

# EE112 - Fall 2016

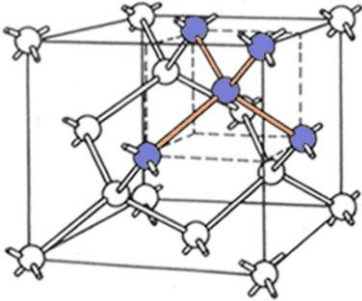
## Analog Integrated Circuits I

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### Silicon: Group IV Element

IA	IIA	IIIB	IVB	VB	VIB	VII B	VIII	IB	IIB	IIIA	IVA	VA	VIA	VIIA	GASES		
1 H 1.00797															2 He 4.0026		
3 Li 6.939	4 Be 9.0122											5 B 10.811	6 C 12.0112	7 N 14.0067	8 O 15.9994	9 F 18.9984	10 Ne 20.183
11 Na 22.9898	12 Mg 24.312											13 Al 26.9815	14 Si 28.086	15 P 30.9738	16 S 32.064	17 Cl 35.453	18 Ar 39.948
19 K 39.102	20 Ca 40.08	21 Sc 44.956	22 Ti 47.90	23 V 50.942	24 Cr 51.996	25 Mn 54.9380	26 Fe 55.847	27 Co 58.9332	28 Ni 58.71	29 Cu 63.54	30 Zn 65.37	31 Ga 69.72	32 Ge 72.59	33 As 74.9216	34 Se 78.96	35 Br 79.909	36 Kr 83.80
37 Rb 85.47	38 Sr 87.62	39 Y 88.905	40 Zr 91.22	41 Nb 92.906	42 Mo 95.94	43 Tc [99]	44 Ru 101.07	45 Rh 102.905	46 Pd 106.4	47 Ag 107.870	48 Cd 112.40	49 In 114.82	50 Sn 118.69	51 Sb 121.75	52 Te 127.60	53 I 126.904	54 Xe 131.30
55 Cs 132.905	56 Ba 137.34	*57 La 138.91	72 Hf 178.49	73 Ta 180.948	74 W 183.85	75 Re 186.2	76 Os 190.2	77 Ir 192.2	78 Pt 195.09	79 Au 196.967	80 Hg 200.59	81 Tl 204.37	82 Pb 207.19	83 Bi 208.980	84 Po (210)	85 At (210)	86 Rn (222)
87 Fr (223)	88 Ra (226)	†89 Ac (227)	104 Rf (261)	105 Db (262)	106 Sg (266)	107 Bh (262)	108 Hs (265)	109 Mt (266)	110 ? (271)	111 ? (272)	112 ? (277)						

## Crystalline Structure (Diamond Cubic)



## 2-D Representation

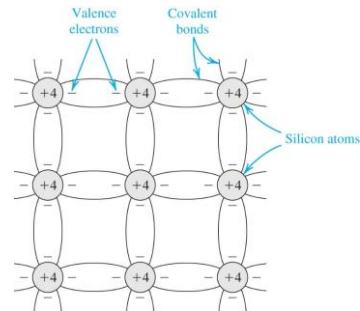


Figure 1.28 Two-dimensional representation of the silicon crystal.

At 0 Kelvin, all electrons are “locked” in covalent bonds (共价键)  
 -> Behave like insulator (绝缘体)

# Electrons and Holes

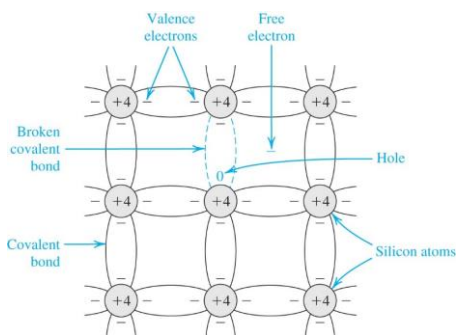


Figure 1.29

### Thermal Generation:

» At room temperature, thermal energy breaks some covalent bonds, creating free electrons and “holes”

### Hole (电洞): empty space left by electron

» Hole “moves” as adjacent electron move into its space

» Treat hole like a positively charged particle

### Recombination (复合)

» Free electrons and holes move randomly through the silicon crystal structure

» Some electrons may fill some of the holes

# Intrinsic (本征) Semiconductor

## ■ Intrinsic semiconductor

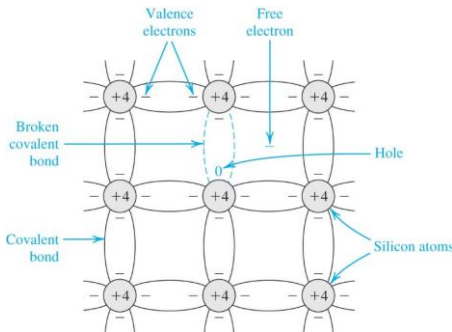


Figure 1.29

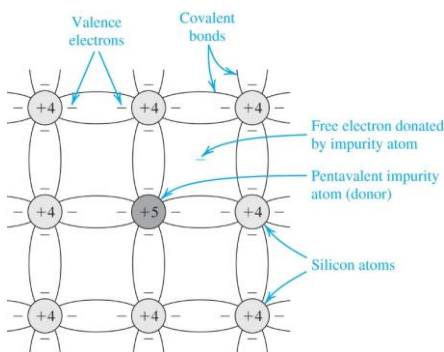
- »  $n = p = n_i$
- »  $n$ : electron concentration [ $\text{cm}^{-3}$ ]
- »  $p$ : hole concentration [ $\text{cm}^{-3}$ ]
- »  $n_i = BT^{3/2}e^{-E_g/(2kT)}$ : intrinsic carrier concentration
  - $B$ : material dependent constant
  - $T$ : temperature in Kelvin
  - $E_g$ : **bandgap(带隙)** energy (=1.12 eV for Si)
  - $k$ : Boltzmann's constant =  $8.62 \times 10^{-5}$  eV/K
- » At room temperature ( $T = 300 \text{ K}$ )

$$n_i = 1.5 \times 10^{10} [\text{cm}^{-3}]$$

**Note:** There are  $5 \times 10^{22}$  atoms/ $\text{cm}^{-3}$ , so the number of free electrons and holes are very small

$$\text{In general, } np = n_i^2$$

# N-Type Semiconductor



**Figure 1.30** A silicon crystal doped by a pentavalent element. Each dopant atom donates a free electron and is thus called a donor. The doped semiconductor becomes  $n$  type.

- Electron concentration can be greatly increased by replacing some Si atoms with P (phosphorus) or As (Arsenic), which have 5 shell electrons (one more than Si).
- P or As are called "**donors (施主)**"
  - $n_n = N_D$  (donor impurity concentration)
  - $p_n = n_i^2/N_D$  where  $n_i = 1.5 \times 10^{10} [\text{cm}^{-3}]$
- Subscript  $n$  refers to  $n$ -type semiconductor ( $n$  stands for "negative", referring to the charge carried by electrons)
- In  $n$ -type semiconductor,  $n_n \gg n_i \gg p_n$
- e.g.,  $N_D = 10^{17} \text{ cm}^{-3}$ ,  $n_n = 10^{17}$ ,  $p_n = 2.2 \times 10^3$
- Electrons are "**majority carriers (多子)**"
- Holes are "**minority carriers (少子)**"

# P-Type Semiconductor

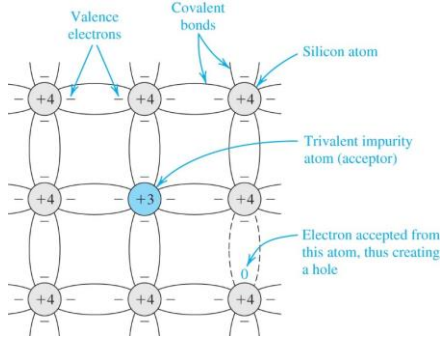


Figure 1.31 A silicon crystal doped with boron, a trivalent impurity. Each dopant atom gives rise to a hole, and the semiconductor becomes p type.

- Hole concentration can be greatly increased by replacing some Si atoms with B (boron), which has 3 shell electrons (one less than Si).
- B is called “**acceptors**” (受主)  
 $p_p = N_A$  (acceptor impurity concentration)  
 $n_p = n_i^2 / N_A$  where  $n_i = 1.5 \times 10^{10} [\text{cm}^{-3}]$
- Subscript  $n$  refers to p-type semiconductor (p stands for “positive”, referring to the charge carried by holes)
- In p-type semiconductor,  $p_p \gg n_i \gg n_p$
- e.g.,  $N_A = 10^{17} \text{ cm}^{-3}$ ,  $p_p = 10^{17}$ ,  $n_p = 2.2 \times 10^3$
- Holes are “majority” carriers
- Electrons are “minority” carriers

## Current in Semiconductor (1): Drift Current (漂移电流)

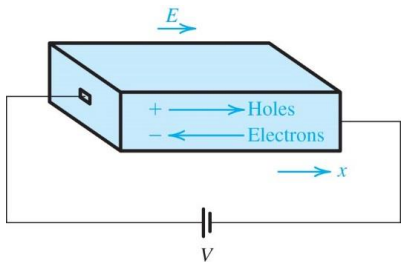


Figure 1.32 An electric field  $E$  established in a bar of silicon causes the holes to drift in the direction of  $E$  and the free electrons to drift in the opposite direction. Both the hole and electron drift currents are in the direction of  $E$ .

When an **electrical field,  $E$** , is applied, holes moves in the direction of  $E$ , while electrons move opposite to  $E$ :

$$\begin{cases} v_{p\text{-drift}} = \mu_p E \\ v_{n\text{-drift}} = -\mu_n E \end{cases}$$

Where,  $\mu_p$  is the **hole mobility (电洞迁移率)**;  $\mu_n$  is the **electron mobility (电子迁移率)**;

In intrinsic Si,  $\mu_n = 1350 \text{ cm}^2/\text{V}\cdot\text{s}$

$$\mu_p = 480 \text{ cm}^2/\text{V}\cdot\text{s} \quad (\text{Note: } \mu_n \approx 2.5\mu_p)$$

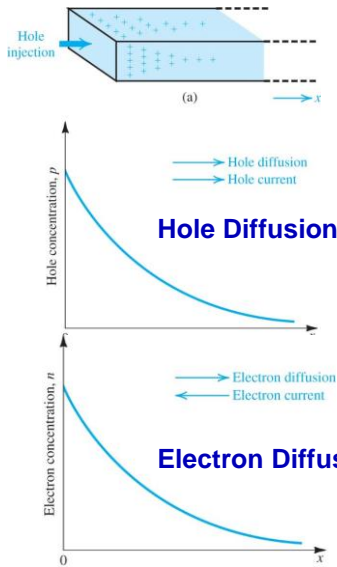
**Current density,  $J$  [A/cm<sup>2</sup>]**

$$J = qp v_{p\text{-drift}} + qn v_{n\text{-drift}} = q(\rho\mu_p + n\mu_n)E = \sigma E$$

where  $\sigma = q(\rho\mu_p + n\mu_n)$  is **conductivity (电导率)** [S/cm]

**Resistivity (电阻率)  $\rho = 1/\sigma$  [Ω-cm]**

# Current in Semiconductor (2) Diffusion Current (扩散电流)



**Diffusion:** particles move from high to low concentrations. As electrons and holes diffuse, currents flow because they carry charges.

$$J_{p-diff} = -qD_p \frac{dp(x)}{dx}$$

$$J_{n-diff} = -(-q)D_n \frac{dn(x)}{dx} = qD_n \frac{dn(x)}{dx}$$

where  $D_p$  and  $D_n$  are hole and electron **diffusion coefficients/diffusivity (扩散率)** [ $\text{cm}^2/\text{s}$ ]

In intrinsic Si,  $D_p = 12 \text{cm}^2/\text{s}$ ,  $D_n = 35 \text{cm}^2/\text{s}$

Total diffusion current density,  $J_{diff}$  [ $\text{A}/\text{cm}^2$ ]

$$J_{diff} = -qD_p \frac{dp(x)}{dx} + qD_n \frac{dn(x)}{dx}$$

## Example

- Given a silicon bar with hole concentration profile

$$p(x) = p_0 e^{-x/L_p}$$

Find the hole-current density at  $x = 0$ . Let  $p_0 = 10^{16}/\text{cm}^3$ ,  $L_p = 1 \text{um}$ , and  $D_p = 12 \text{cm}^2/\text{s}$ .

If the cross-sectional area of the bar is  $100 \text{um}^2$ , find the current  $I_p$ .

# Einstein Relationship

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T = \frac{kT}{q}$$

- $V_T$ : Thermal voltage. At room temperature,  $V_T = 26$  mV
  
- Proof: