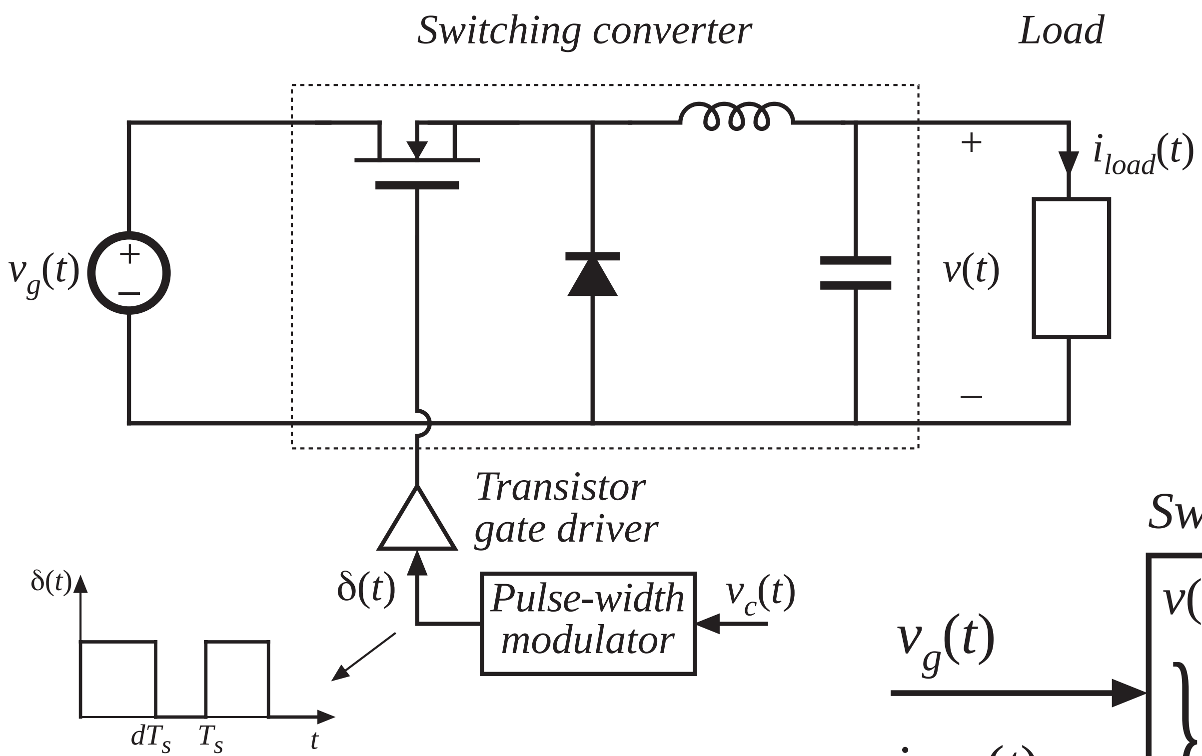
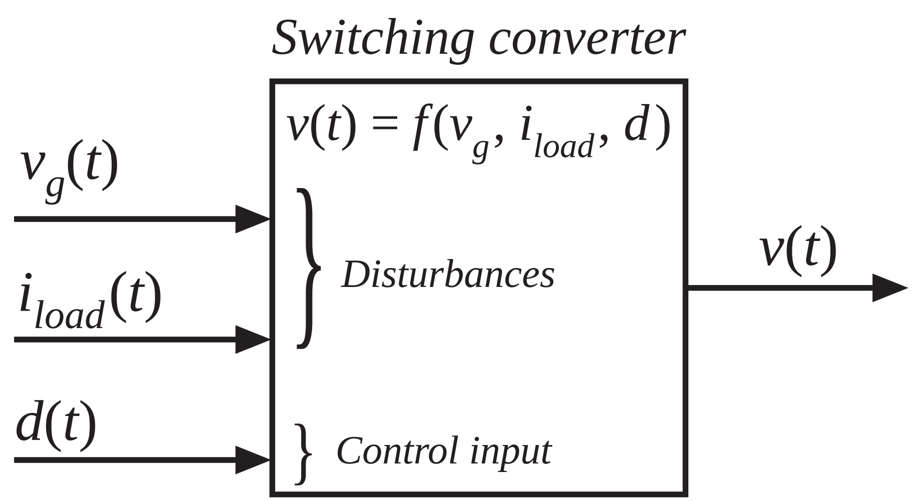
Controller Design

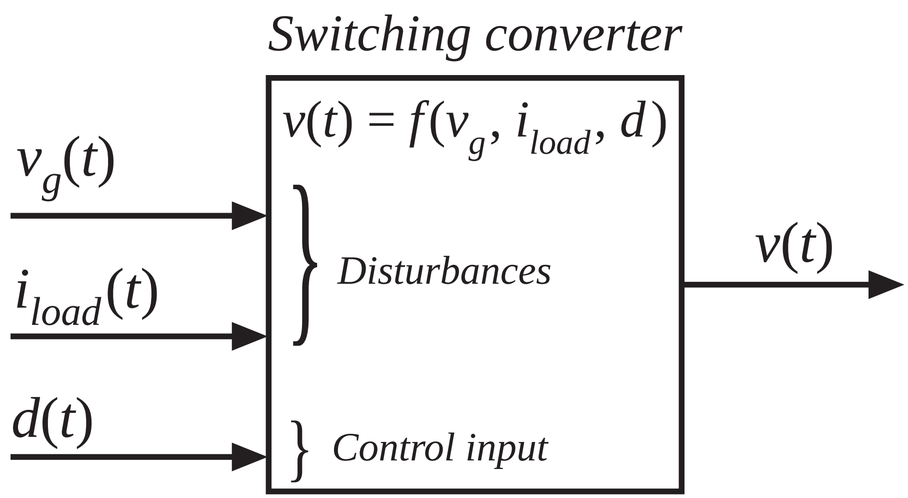


Output voltage of a switching converter depends on duty cycle d, input voltage v_g, and load current i_{load}.



The dc regulator application

Objective: maintain constant output voltage $v(t) = V$, in spite of disturbances in $v_g(t)$ and $i_{load}(t)$.



Input source variation

Typical variation in $v_g(t)$: 100Hz or 120Hz ripple, produced by rectifier circuit.

Load variation

Load current variations: a significant step-change in load current, such as from 50% to 100% of rated value, may be applied.

A typical output voltage regulation specification: $5\text{V} \pm 0.1\text{V}$.

Circuit elements are constructed to some specified tolerance. In high volume manufacturing of converters, all output voltages must meet specifications.

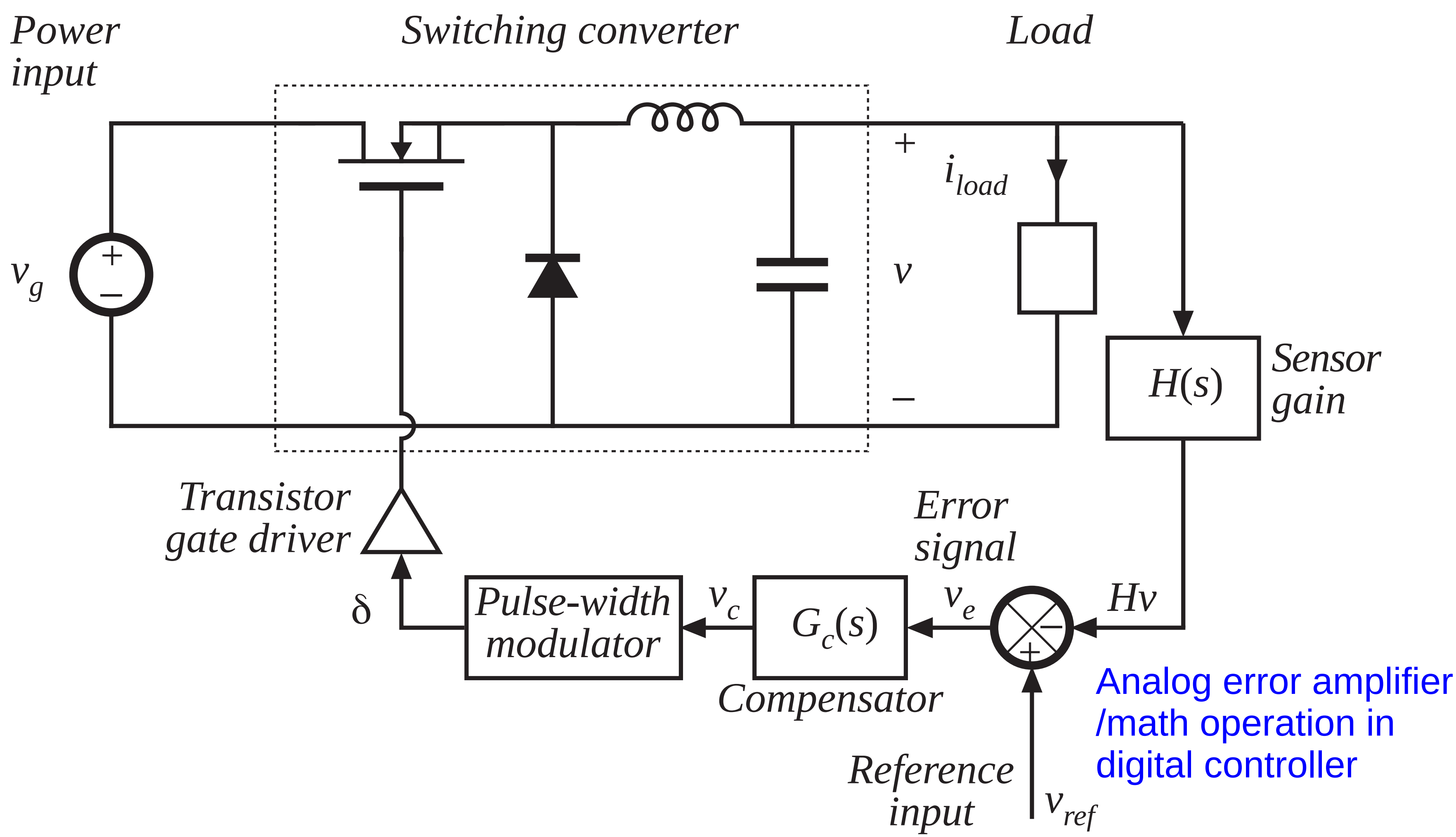
converter need to resist the disturbances and maintain a constant output

The dc regulator application

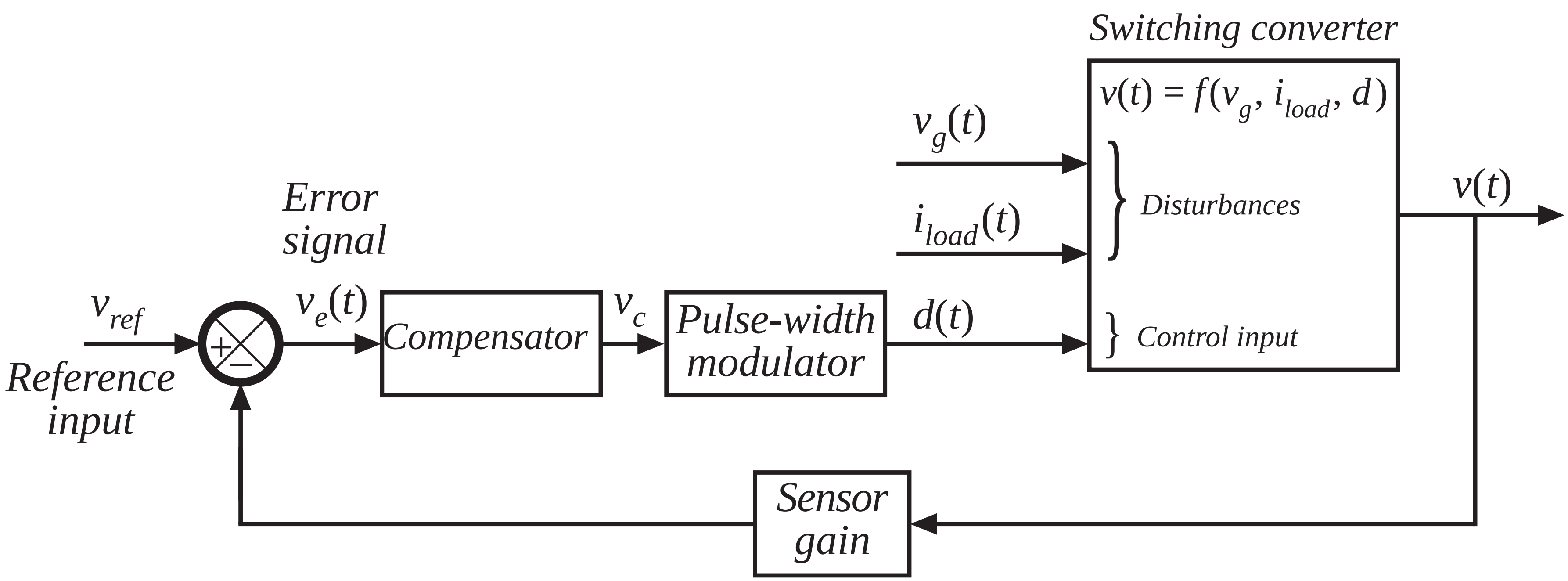
So we cannot expect to set the duty cycle to a single value, and obtain a given constant output voltage under all conditions.

Negative feedback: build a circuit that automatically adjusts the duty cycle as necessary, to obtain the specified output voltage with high accuracy, regardless of disturbances or component tolerances.

Negative feedback:
a switching regulator system

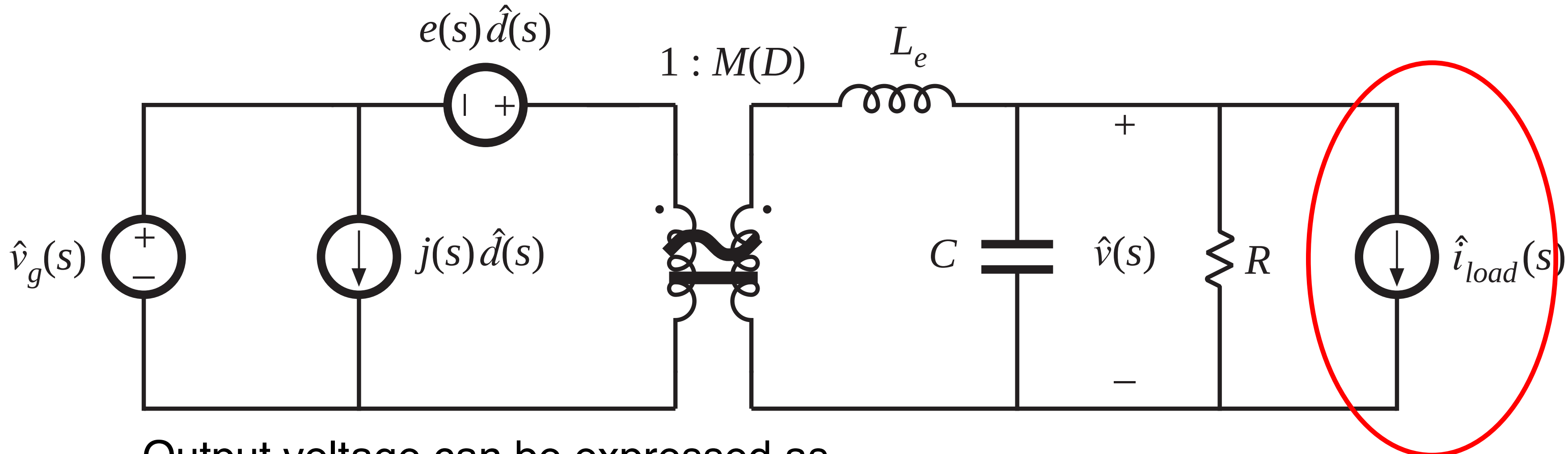


Negative feedback



9.2. Effect of negative feedback on the network transfer functions

Small signal model: open-loop converter



Output voltage can be expressed as

$$\hat{v}(s) = G_{vd}(s)\,\hat{d}(s) + G_{vg}(s)\,\hat{v}_g(s) - Z_{out}(s)\,\hat{i}_{load}(s)$$

where

$$G_{vd}(s) = \left.\frac{\hat{v}(s)}{\hat{d}(s)}\right|_{\substack{\hat{v}_g = 0 \\ \hat{i}_{load} = 0}} \qquad G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\substack{\hat{d} = 0 \\ \hat{i}_{load} = 0}} \qquad Z_{out}(s) = -\left.\frac{\hat{v}(s)}{\hat{i}_{load}(s)}\right|_{\substack{\hat{d} = 0 \\ \hat{v}_g = 0}}$$

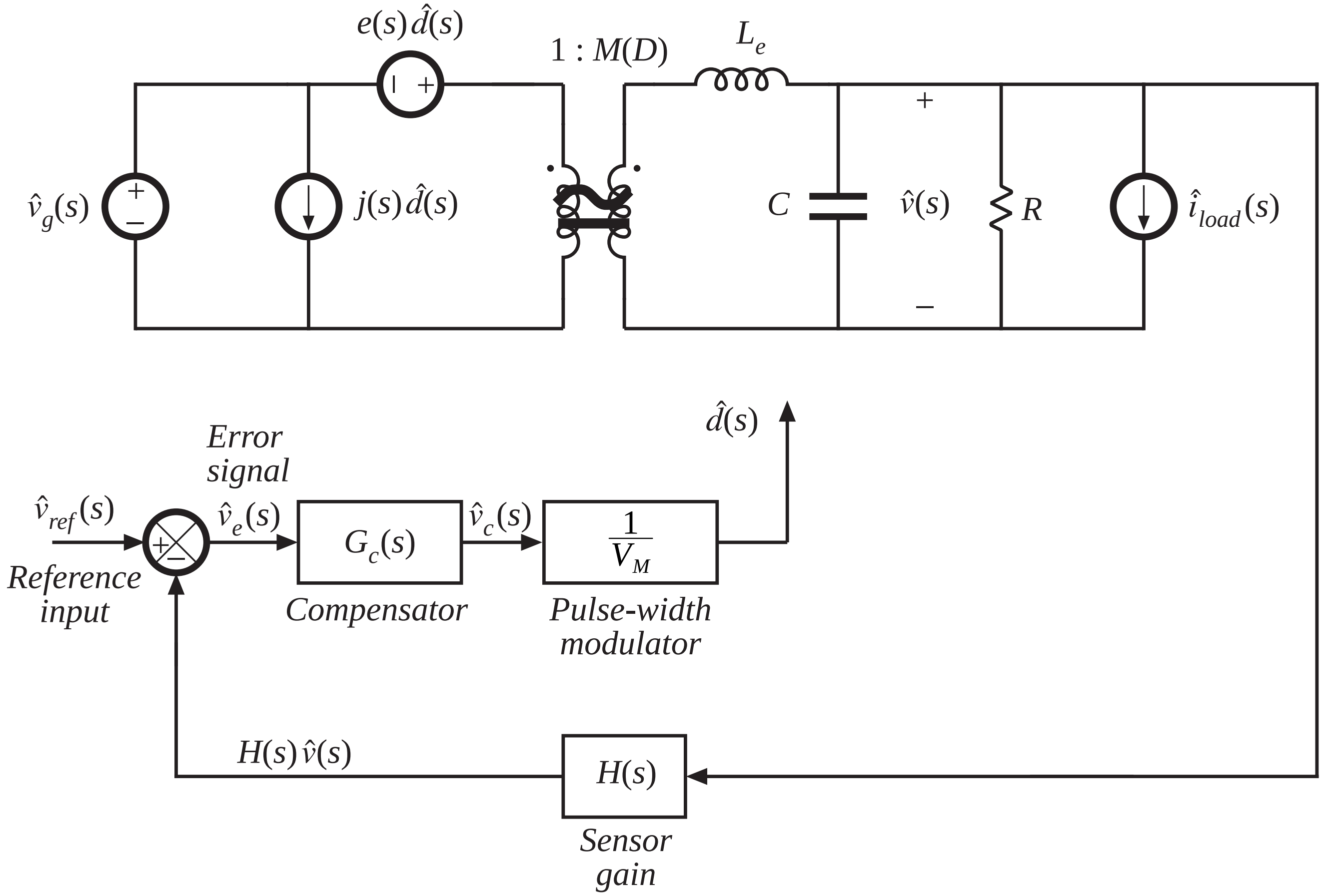
Voltage regulator system small-signal model

- *Use small-signal converter model*
- *Perturb and linearize remainder of feedback loop:*

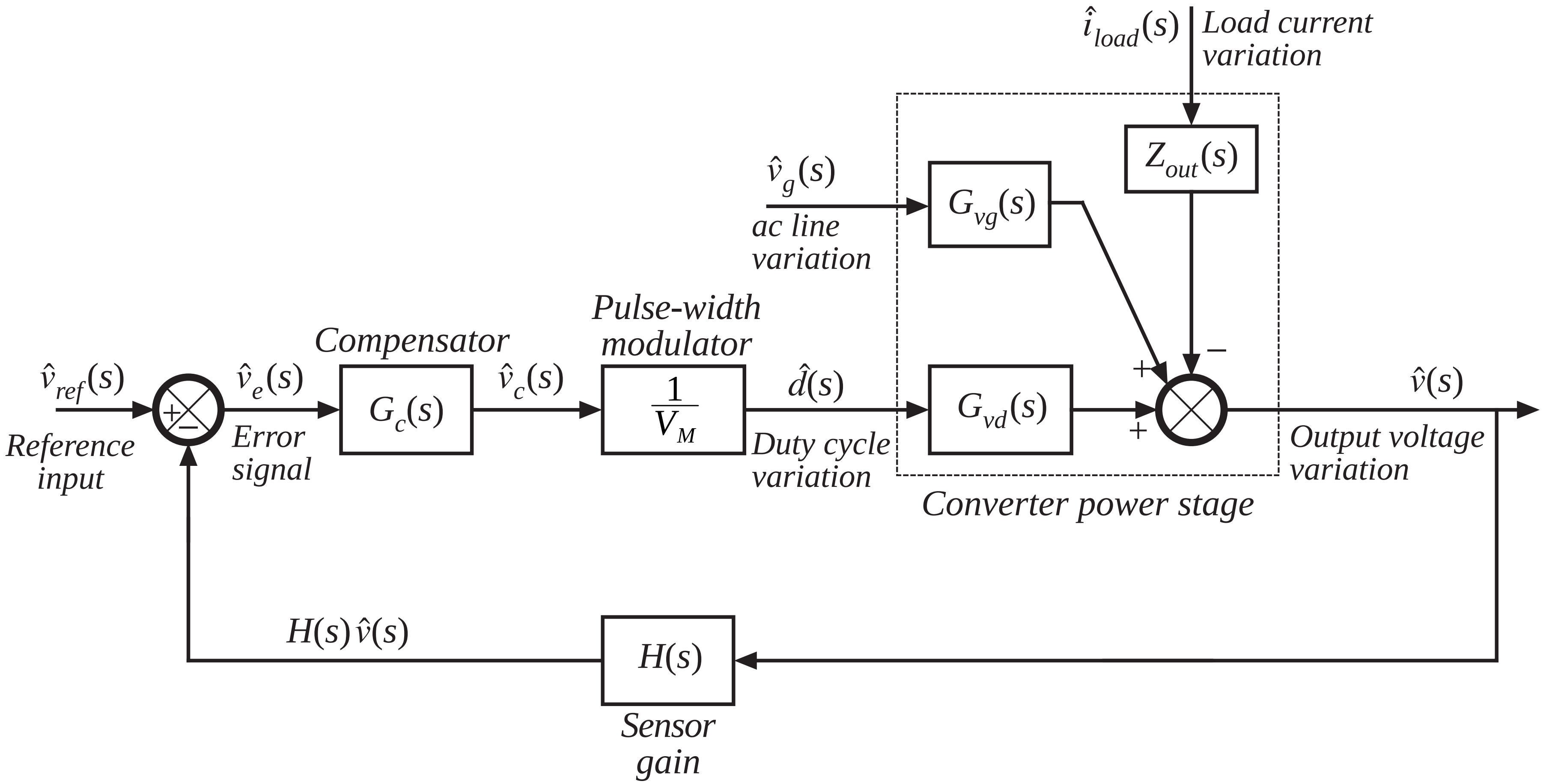
$$v_{ref}(t) = V_{ref} + \hat{v}_{ref}(t)$$

$$v_e(t) = V_e + \hat{v}_e(t)$$

etc.



Regulator system small-signal block diagram



Solution of block diagram

Manipulate block diagram to solve for $\hat{v}(s)$. Result is

Rectro: Y(s)/X(s) = H'(s)/[1+βH'(s)]

$$\hat{v} = \hat{v}_{ref} \frac{G_c G_{vd} / V_M}{1 + H G_c G_{vd} / V_M} + \hat{v}_g \frac{G_{vg}}{1 + H G_c G_{vd} / V_M} - \hat{i}_{load} \frac{Z_{out}}{1 + H G_c G_{vd} / V_M}$$

where, H'(s) open loop gain; β = feedback gain

which is of the form

$$\hat{v} = \hat{v}_{ref} \frac{1}{H} \frac{T}{1 + T} + \hat{v}_g \frac{G_{vg}}{1 + T} - \hat{i}_{load} \frac{Z_{out}}{1 + T}$$

$$\textit{with} \quad T(s) = H(s)\, G_c(s)\, G_{vd}(s) / V_M \quad = \text{"loop gain"}$$

where, T(s) = βH'(s)

Loop gain $T(s)$ = products of the gains around the negative feedback loop.

9.2.1. Feedback reduces the transfer functions from disturbances to the output

Original (open-loop) line-to-output transfer function:

$$G_{vg}(s) = \left. \frac{\hat{v}(s)}{\hat{v}_g(s)} \right|_{\substack{\hat{d} = 0 \\ \hat{i}_{load} = 0}}$$

With addition of negative feedback, the line-to-output transfer function becomes:

$$\left. \frac{\hat{v}(s)}{\hat{v}_g(s)} \right|_{\substack{\hat{v}_{ref} = 0 \\ \hat{i}_{load} = 0}} = \frac{G_{vg}(s)}{1 + T(s)}$$

Feedback reduces the line-to-output transfer function by a factor of

$$\frac{1}{1 + T(s)}$$

If $T(s)$ is large in magnitude, then the line-to-output transfer function becomes small.

Closed-loop output impedance

Original (open-loop) output impedance:

$$Z_{out}(s) = -\left.\frac{\hat{v}(s)}{\hat{i}_{load}(s)}\right|_{\substack{\hat{d}=0 \\ \hat{v}_g=0}}$$

With addition of negative feedback, the output impedance becomes:

$$\left.\frac{\hat{v}(s)}{-\hat{i}_{load}(s)}\right|_{\substack{\hat{v}_{ref}=0 \\ \hat{v}_g=0}} = \frac{Z_{out}(s)}{1+T(s)}$$

Feedback reduces the output impedance by a factor of

$$\frac{1}{1+T(s)}$$

If $T(s)$ is large in magnitude, then the output impedance is greatly reduced in magnitude.

9.2.2. Feedback causes the transfer function from the reference input to the output to be insensitive to variations in the gains in the forward path of the loop

Closed-loop transfer function from $\hat{v}_{ref}$ to $\hat{v}(s)$ is:

$$\left.\frac{\hat{v}(s)}{\hat{v}_{ref}(s)}\right|_{\substack{\hat{v}_g = 0 \\ \hat{i}_{load} = 0}} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)}$$

If the loop gain is large in magnitude, i.e., $\| T \| >> 1$, then $(1+T) \approx T$ and $T/(1+T) \approx T/T = 1$. The transfer function then becomes

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} \approx \frac{1}{H(s)}$$

which is independent of the gains in the forward path of the loop.

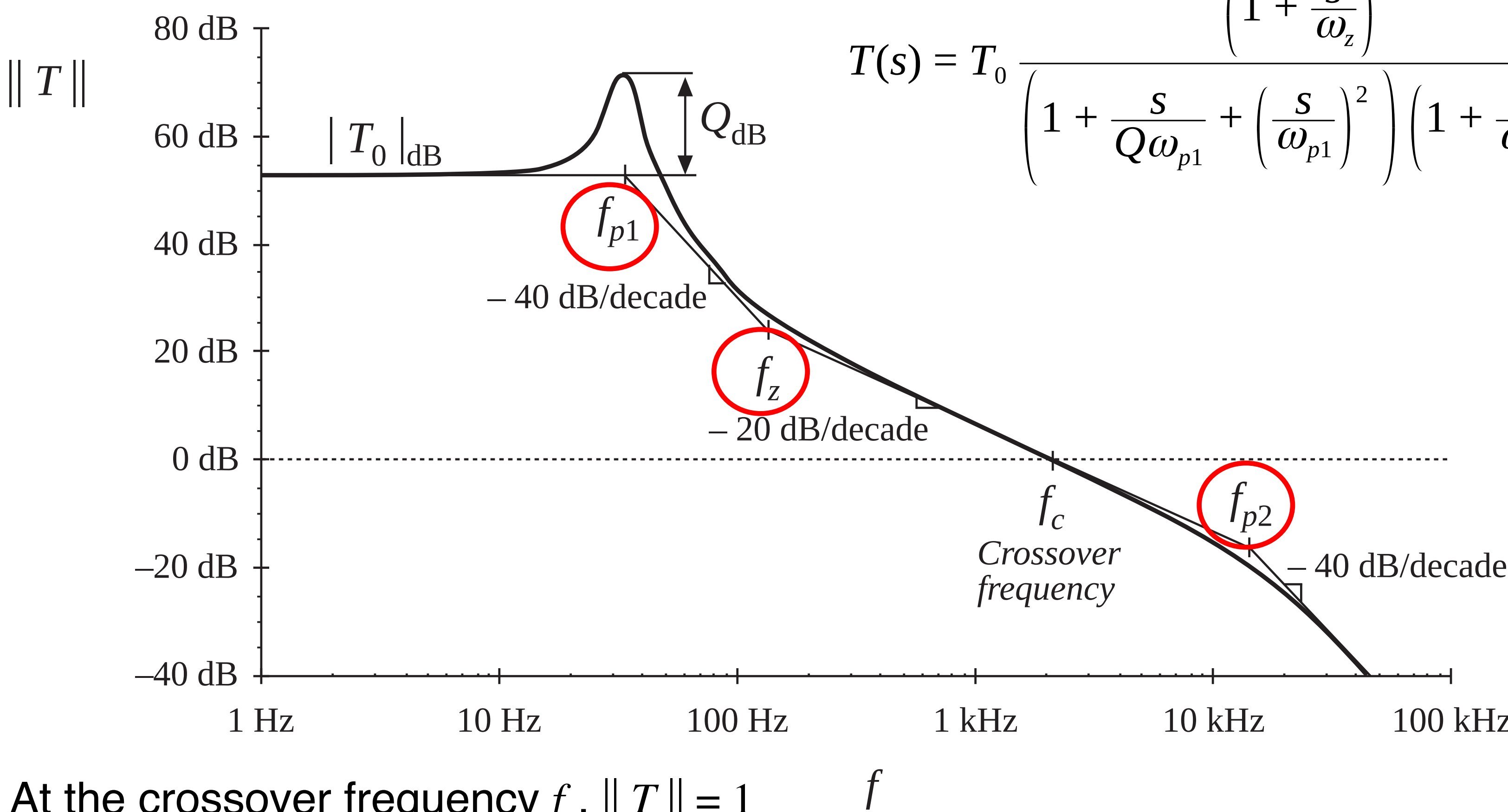
This result applies equally well to dc values:

$$\frac{V}{V_{ref}} = \frac{1}{H(0)} \frac{T(0)}{1 + T(0)} \approx \frac{1}{H(0)}$$

9.3. Construction of the important quantities $1/(1+T)$ and $T/(1+T)$

Example

$$T(s) = T_0 \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{Q\omega_{p1}} + \left(\frac{s}{\omega_{p1}}\right)^2\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$



At the crossover frequency f_c, $\| T \| = 1$

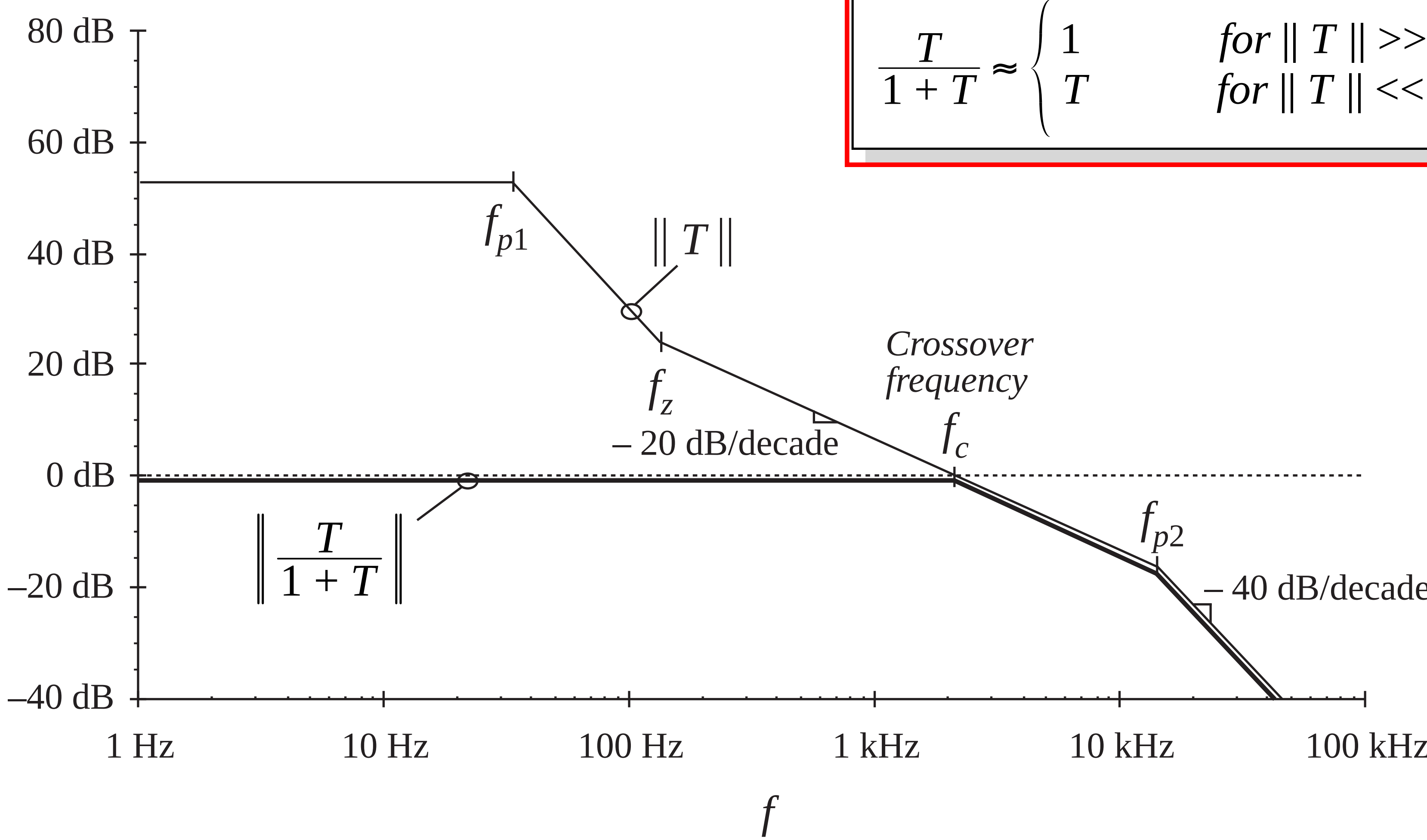
Approximating $1/(1+T)$ and $T/(1+T)$

$$\frac{T}{1+T} \approx \begin{cases} 1 & \text{for } \| T \| >> 1 \\ T & \text{for } \| T \| << 1 \end{cases}$$

$$\frac{1}{1+T(s)} \approx \begin{cases} \dfrac{1}{T(s)} & \text{for } \| T \| >> 1 \\ 1 & \text{for } \| T \| << 1 \end{cases}$$

Example: construction of $T/(1+T)$

$$\frac{T}{1+T} \approx \begin{cases} 1 & \text{for } \| T \| >> 1 \\ T & \text{for } \| T \| << 1 \end{cases}$$



Example: analytical expressions for approximate reference to output transfer function

for f < f_c, $\|T\| \gg 1$, therefore,

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)} \approx \frac{1}{H(s)}$$

The output follows the reference according to the ideal gain 1/H(s)

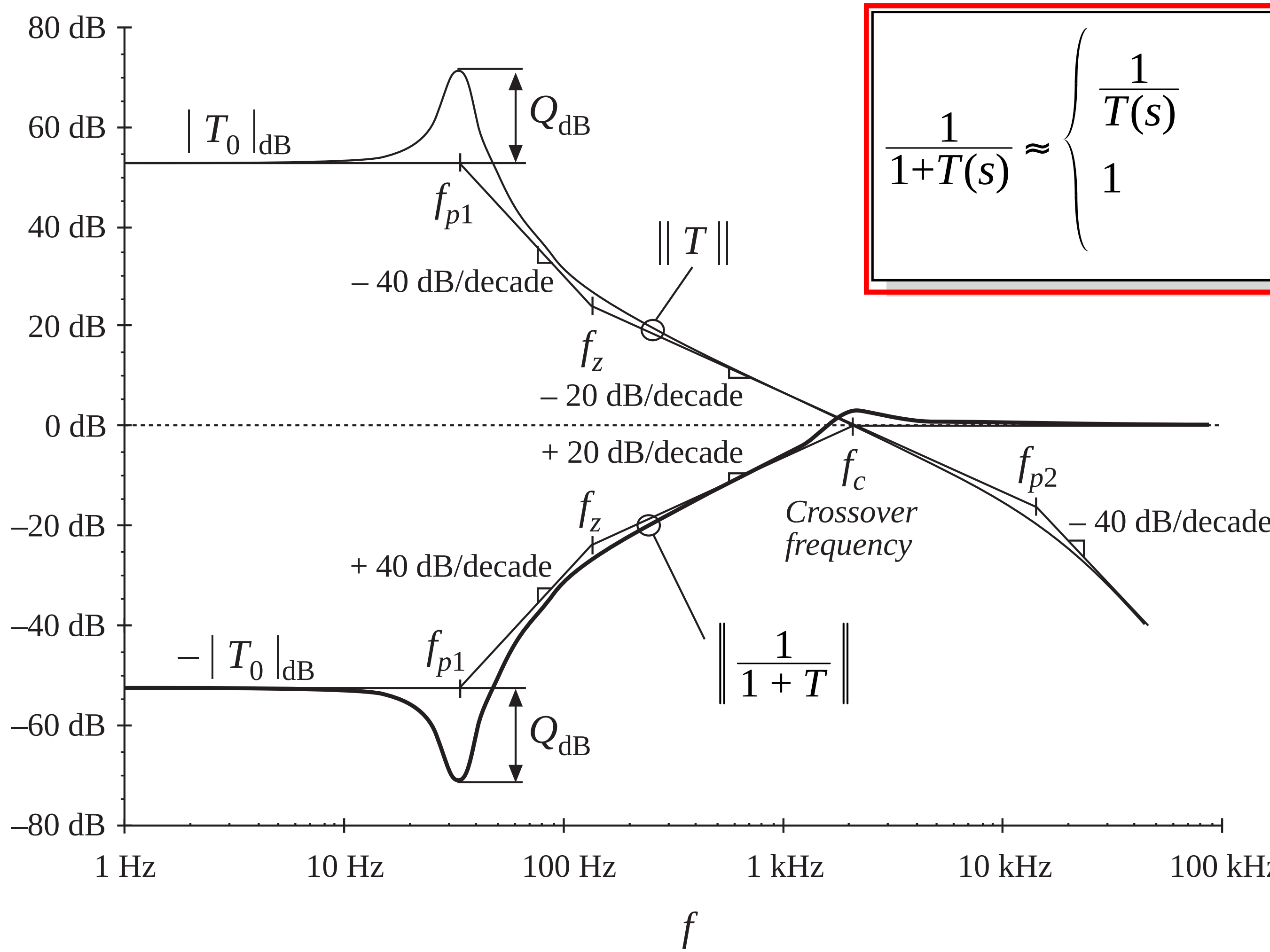
At frequencies above the crossover frequency, $\| T \| < 1$. The quantity $T/(1+T)$ then has magnitude approximately equal to 1, and we obtain

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)} \approx \frac{T(s)}{H(s)} = \frac{G_c(s)G_{vd}(s)}{V_M}$$

This coincides with the open-loop transfer function from the reference to the output. At frequencies where $\| T \| < 1$, the loop has essentially no effect on the transfer function from the reference to the output.

Same example: construction of 1/(1+T)

$$\frac{1}{1+T(s)} \approx \begin{cases} \frac{1}{T(s)} & \text{for } \| T \| >> 1 \\ 1 & \text{for } \| T \| << 1 \end{cases}$$



Interpretation: how the loop rejects disturbances

Below the crossover frequency: $f < f_c$ and $\| T \| > 1$

Then $1/(1+T) \approx 1/T$, and disturbances are reduced in magnitude by $1/\| T \|$

Above the crossover frequency: $f > f_c$ and $\| T \| < 1$

Then $1/(1+T) \approx 1$, and the feedback loop has essentially no effect on disturbances

$$\frac{1}{1+T(s)} \approx \begin{cases} \frac{1}{T(s)} & \text{for } \| T \| >> 1 \\ 1 & \text{for } \| T \| << 1 \end{cases}$$

Terminology: open-loop vs. closed-loop

Original transfer functions, before introduction of feedback ("open-loop transfer functions"):

$$G_{vd}(s) \qquad G_{vg}(s) \qquad Z_{out}(s)$$

Upon introduction of feedback, these transfer functions become ("closed-loop transfer functions"):

$$\frac{1}{H(s)}\frac{T(s)}{1+T(s)} \qquad \frac{G_{vg}(s)}{1+T(s)} \qquad \frac{Z_{out}(s)}{1+T(s)}$$

The loop gain:

$$T(s)$$

9.4. Stability

Even though the original open-loop system is stable, the closed-loop transfer functions can be unstable and contain right half-plane poles. Even when the closed-loop system is stable, the transient response can exhibit undesirable ringing and overshoot, due to the high Q -factor of the closed-loop poles in the vicinity of the crossover frequency.

The feedback might destabilize the system: the denominator $(1+T(s))$ terms in the closed-loop transfer functions contain roots in the right half-plane (i.e., with positive real parts).

Suppose: T(s) = N(s)/D(s), then,

$$\frac{T(s)}{1+T(s)} = \frac{\frac{N(s)}{D(s)}}{1+\frac{N(s)}{D(s)}} = \frac{N(s)}{N(s)+D(s)}$$

$$\frac{1}{1+T(s)} = \frac{1}{1+\frac{N(s)}{D(s)}} = \frac{D(s)}{N(s)+D(s)}$$

- Could evaluate stability by evaluating $N(s) + D(s)$, then factoring to evaluate roots.

However, This is a lot of work, and is not very illuminating.

Determination of stability directly from $T(s)$

Think about what we covered in the last lecture!

- Nyquist stability theorem: general result.
- A special case of the Nyquist stability theorem: the phase margin test

  Allows determination of closed-loop stability (i.e., whether $1/(1+T(s))$ contains RHP poles) directly from the magnitude and phase of $T(s)$.

  A good design tool: yields insight into how $T(s)$ should be shaped, to obtain good performance in transfer functions containing $1/(1+T(s))$ terms.

9.4.1. The phase margin test

A test on $T(s)$, to determine whether $1/(1+T(s))$ contains RHP poles.

The crossover frequency f_c is defined as the frequency where

$$\| T(j2\pi f_c) \| = 1 \Rightarrow 0\text{dB}$$

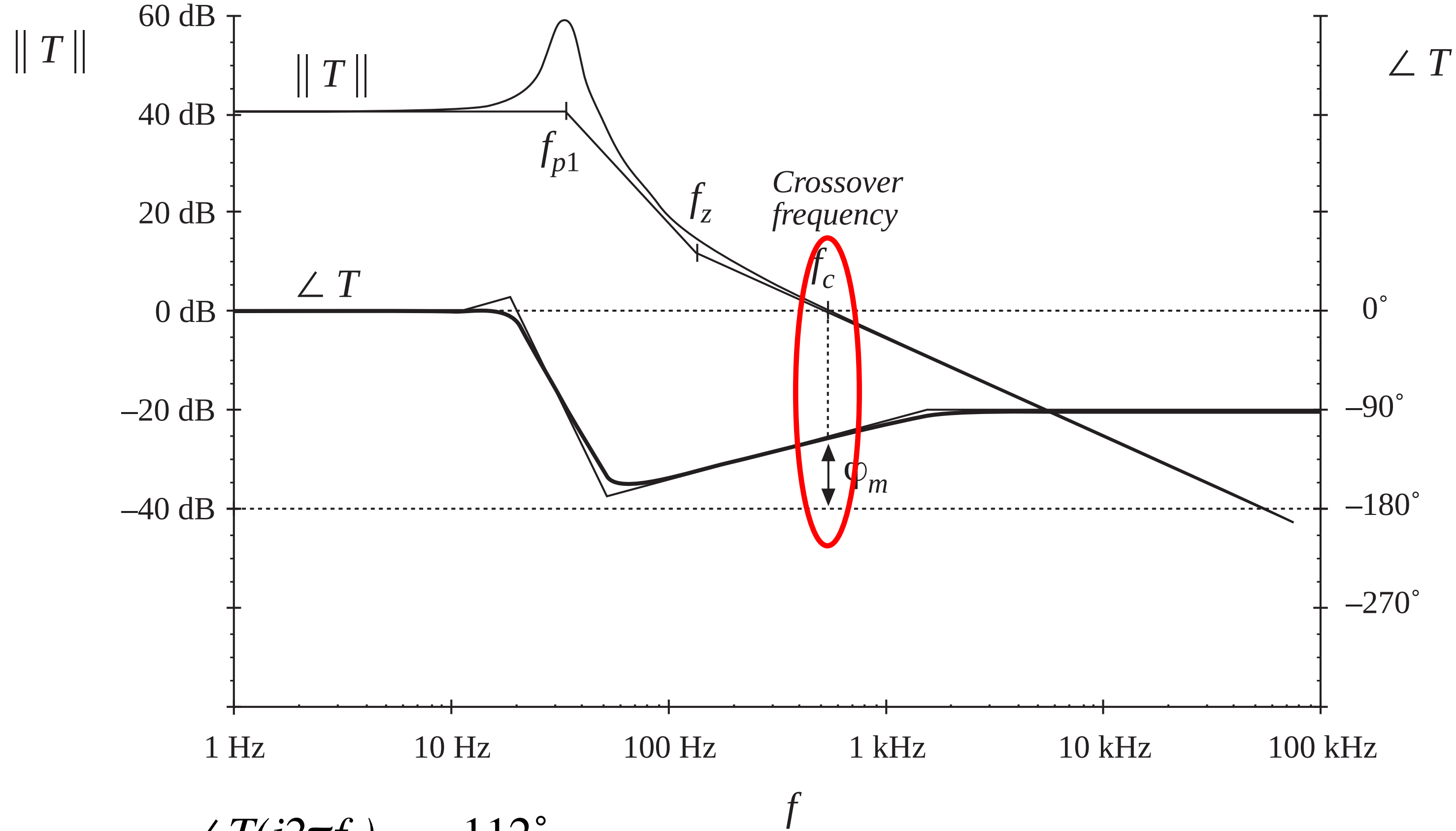
The phase margin φ_m is determined from the phase of $T(s)$ at f_c , as follows:

$$\varphi_m = 180^\circ + \angle T(j2\pi f_c)$$

If there is exactly one crossover frequency, and if $T(s)$ contains no RHP poles, then

> the quantities $T(s)/(1+T(s))$ and $1/(1+T(s))$ contain no RHP poles whenever the phase margin φ_m is positive.

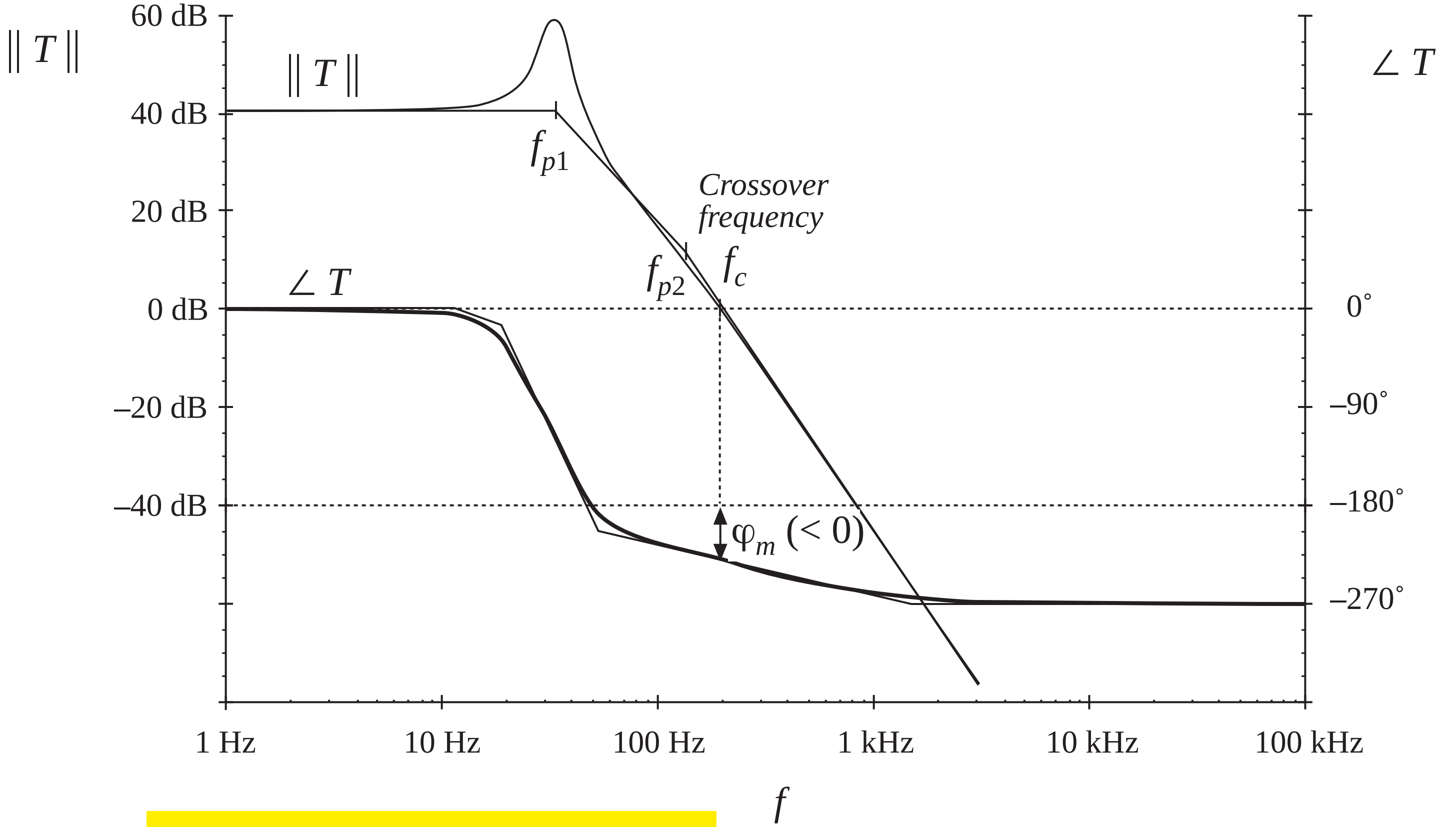
Example: a loop gain leading to a stable closed-loop system



$$\angle T(j2\pi f_c) = -112^\circ$$

$$\varphi_m = 180^\circ - 112^\circ = +68^\circ$$

Example: a loop gain leading to an unstable closed-loop system



$$\angle T(j2\pi f_c) = -230^\circ$$

$$\varphi_m = 180^\circ - 230^\circ = -50^\circ$$

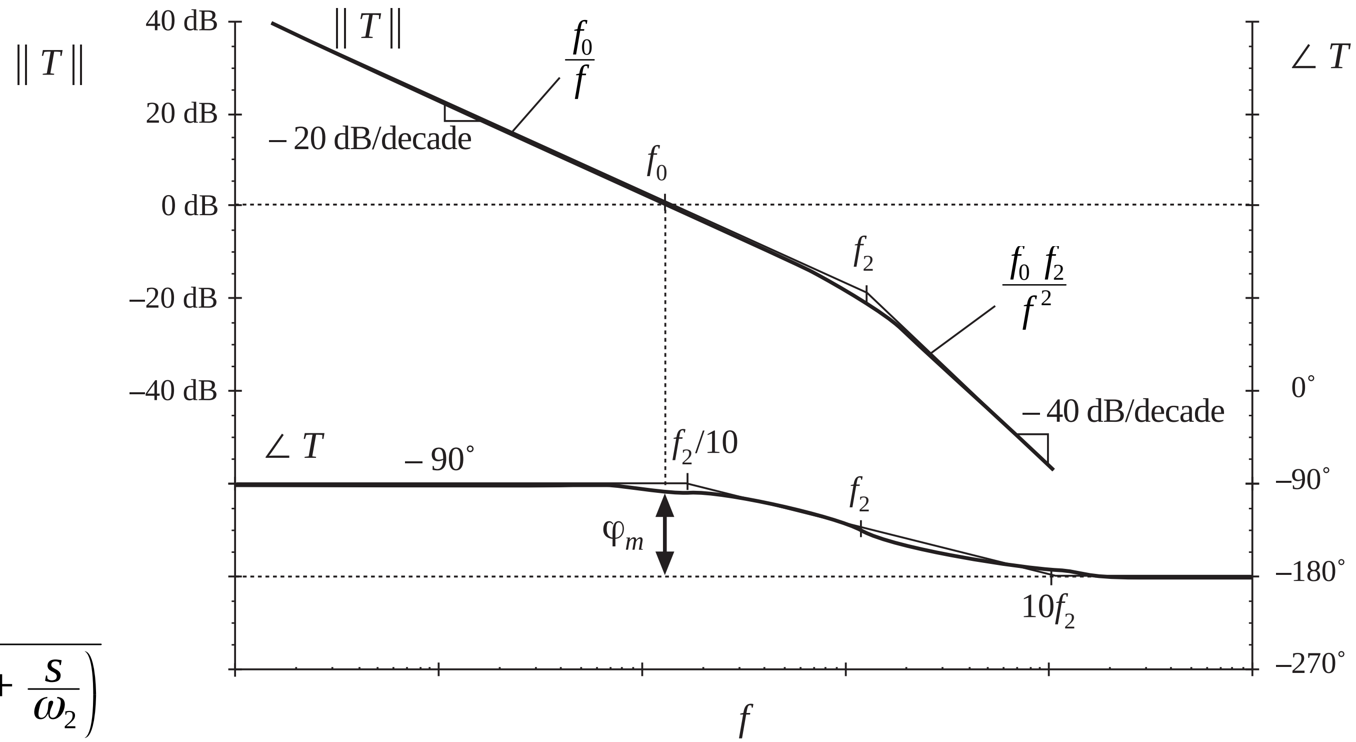
9.4.2. The relation between phase margin and closed-loop damping factor

How much phase margin is required?

A small positive phase margin leads to a stable closed-loop system having complex poles near the crossover frequency with high Q. The transient response exhibits overshoot and ringing.

Increasing the phase margin reduces the Q. Obtaining real poles, with no overshoot and ringing, requires a large phase margin.

A simple second-order system



$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

f_0: magnitude crossover frequency

Closed-loop response

If

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

Then

$$\frac{T(s)}{1 + T(s)} = \frac{1}{1 + \frac{1}{T(s)}} = \frac{1}{1 + \frac{s}{\omega_0} + \frac{s^2}{\omega_0 \omega_2}}$$

or,

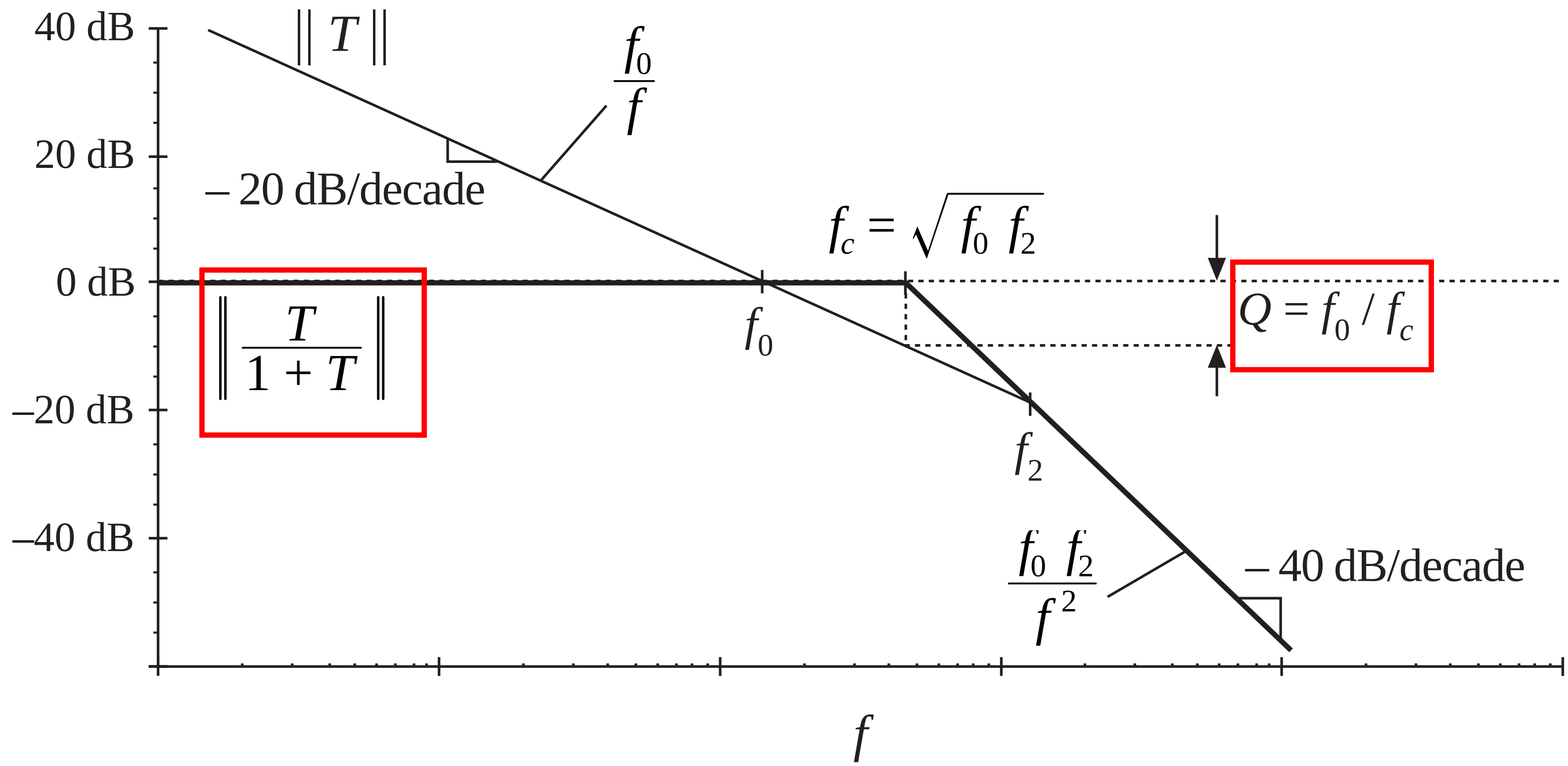
$$\frac{T(s)}{1 + T(s)} = \frac{1}{1 + \frac{s}{Q\omega_c} + \left(\frac{s}{\omega_c}\right)^2}$$

where

$$\omega_c = \sqrt{\omega_0 \omega_2} = 2\pi f_c \qquad Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$$

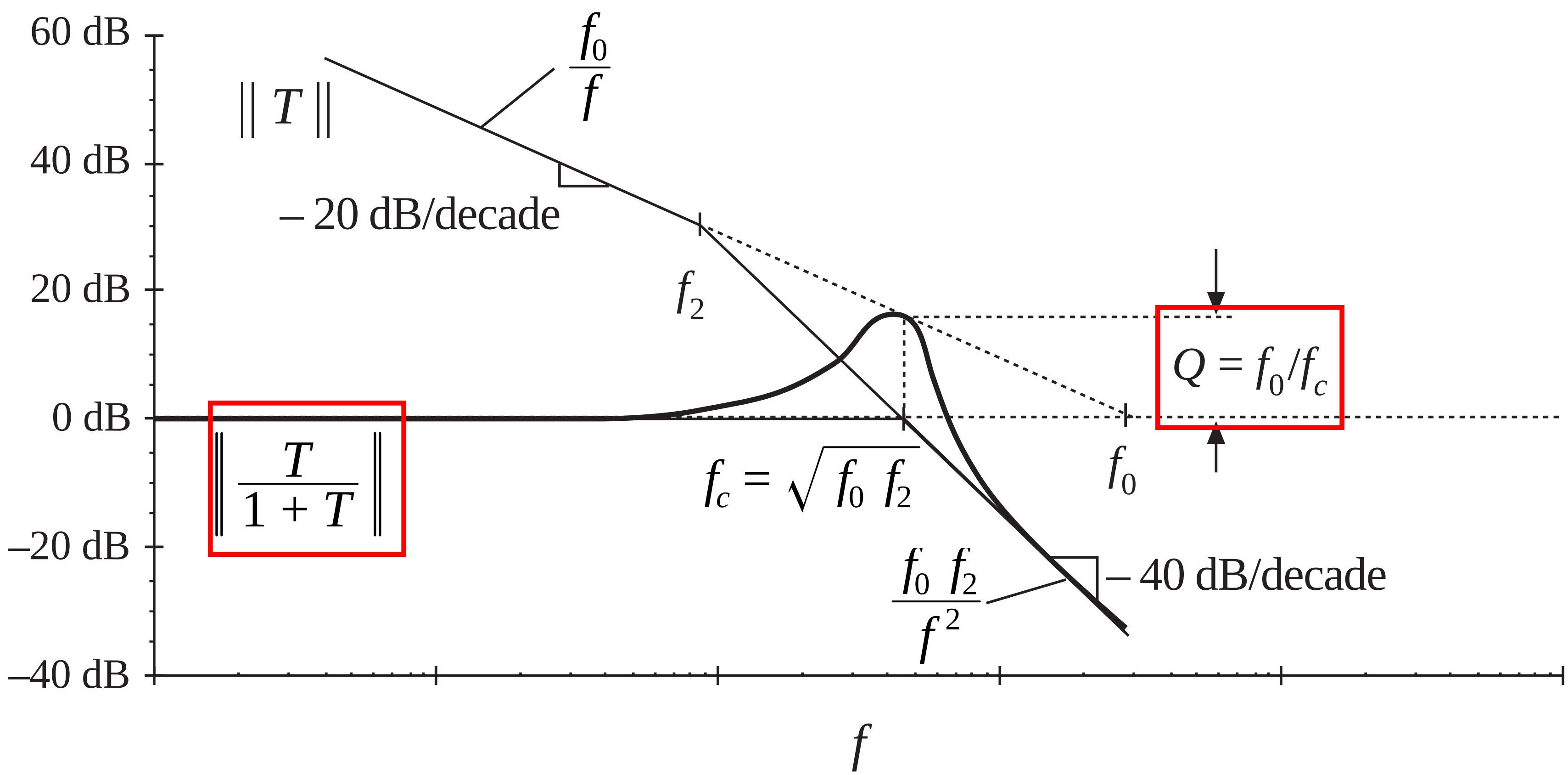
Low-Q case

$$Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}} \qquad Q\,\omega_c = \omega_0 \qquad \frac{\omega_c}{Q} = \omega_2$$



High-Q case

$$\omega_c = \sqrt{\omega_0 \omega_2} = 2\pi f_c \qquad Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$$



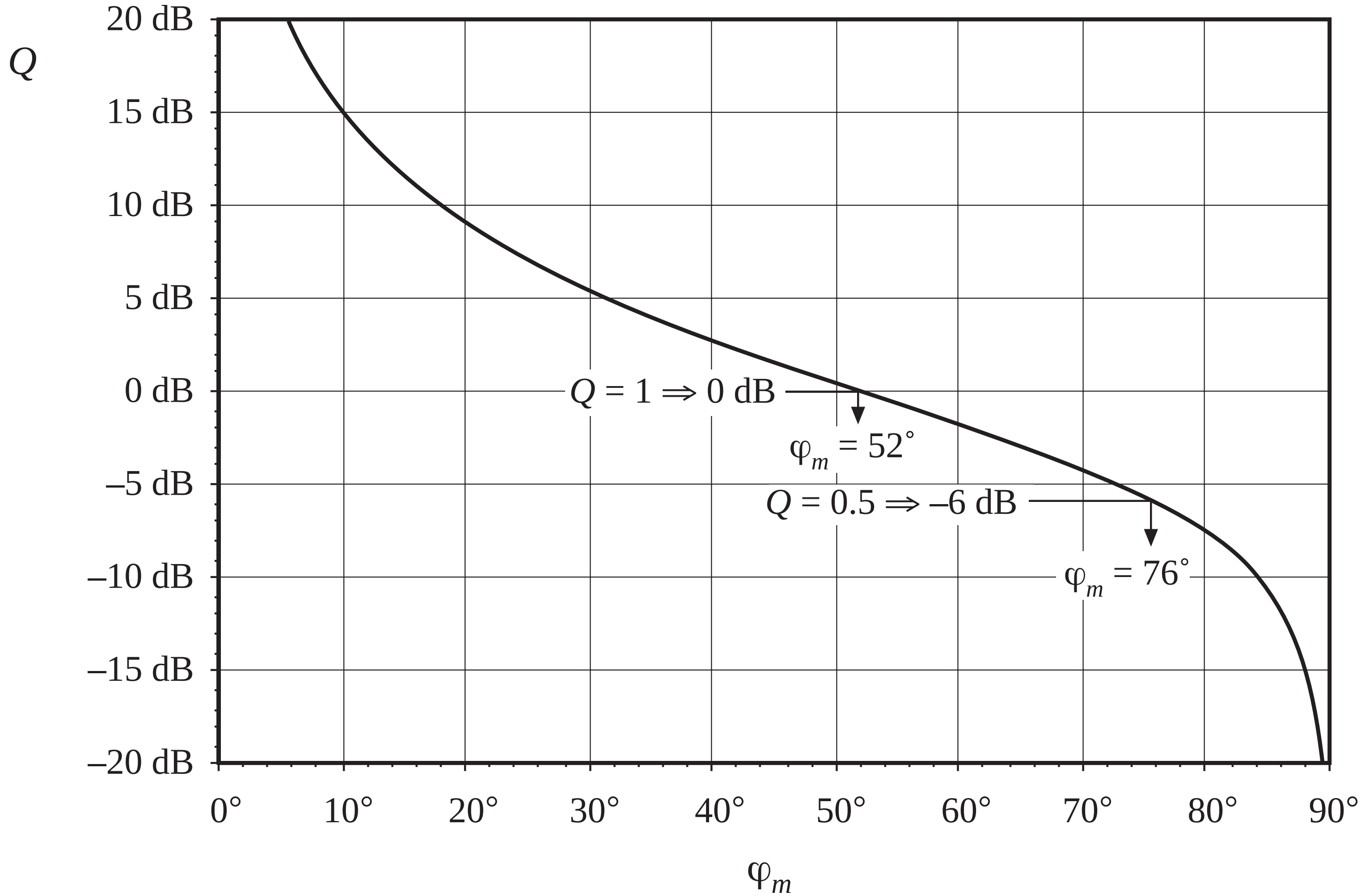
Q vs. φ_m

Solve for exact crossover frequency, evaluate phase margin, express as function of φ_m. Result is:

$$Q = \frac{\sqrt{\cos\varphi_m}}{\sin\varphi_m}$$

$$\varphi_m = \tan^{-1}\sqrt{\frac{1 + \sqrt{1 + 4Q^4}}{2Q^4}}$$

Q vs. φ_m



Q
20 dB
15 dB
10 dB
5 dB
0 dB
–5 dB
–10 dB
–15 dB
–20 dB
Q = 1 ⇒ 0 dB
φm = 52˚
Q = 0.5 ⇒ –6 dB
φm = 76˚
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
φm

9.4.3. Transient response vs. damping factor

Unit-step response of second-order system $T(s)/(1+T(s))$

$$\hat{v}(t) = 1 + \frac{2Q\, e^{-\omega_c t/2Q}}{\sqrt{4Q^2 - 1}} \sin\left[\frac{\sqrt{4Q^2-1}}{2Q}\,\omega_c\, t + \tan^{-1}\left(\sqrt{4Q^2-1}\right)\right] \qquad Q > 0.5$$

$$\hat{v}(t) = 1 - \frac{\omega_2}{\omega_2 - \omega_1}\, e^{-\omega_1 t} - \frac{\omega_1}{\omega_1 - \omega_2}\, e^{-\omega_2 t} \qquad Q < 0.5$$

$$\omega_1,\ \omega_2 = \frac{\omega_c}{2Q}\left(1 \pm \sqrt{1 - 4Q^2}\right)$$

For $Q > 0.5$, the peak value is

$$peak\ \hat{v}(t) = 1 + e^{-\pi/\sqrt{4Q^2 - 1}}$$

Transient response vs. damping factor