Reduced-Order Model for Inductive Power Transfer Systems

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Abstract—As a DC-DC converter, an inductive power transfer (IPT) system requires a feedback control to stabilize the output. An accurate and simple small signal model is important for IPT systems to evaluate the control performance. In the past years, the extended describing function (EDF) is powerful to address the modeling issue for resonant converter. However, the high-order resonant tank of IPT would lead to a complicated model if EDF is directly applied. In order to simplify the model, this paper explores a circuit-based method to reduce the order for both series and parallel resonant circuits. An example LCC-C compensated system is used to explain the concept. This general simplification can be extended for any other high-order IPT systems.

Index Terms—Inductive power transfer, equivalent circuit model, extended describing function, reduce order model

I. INTRODUCTION

Inductive power transfer (IPT) can deliver power from a coil to another coil without any contact. This kind of power transfer mechanism is convenient and safe [1]–[3]. Recently, IPT has been widely used for charging various devices, such as LED, medical implants, electric vehicles and cell phones [4]–[6]. This new power transfer mechanism has the capacity to revolutionize many manufacturing processes.

In order to boost the power transfer ability and minimize the voltampere (VA) rating, four basic compensations (series-series, parallel-series, series-parallel, and parallel-parallel) were proposed [7], [8]. Recently, many high-order compensations have been proposed for constant voltage or constant current applications [9]. These high-order compensated IPT systems can improve the controllability and design freedom. For example, a two-stage fast charger is proposed in [10] to give a constant output voltage with the help of a front-end regulation stage. Similar concept could be also found in EV chargers, where the front-end PFC stage is used for final regulation [11]. Using such control scheme, it is important to derive small-signal transfer function for the IPT stage, and then the controller in the primary side can be designed accordingly to improve the system dynamics.

The IPT system is actually a high-order resonant converter, and thus the modeling approach for resonant converter can be naturally introduced to the IPT system. Average concept [12] is a popular modeling approach for pulse width modulation (PWM) converter. This concept is simple and mature for controller design [13]. However, in resonant converters, the average of some state variables are zero, which makes average model invalid. Another challenge for modeling is that the resonant converter contains nonlinear parts such as inverter and rectifier. These nonlinear parts increase modeling difficulty. In order to overcome these challenges, some methods are presented, i.e., generalized state space averaging (GSSA) [14], extended describing function (EDF) [15] and data sampling model [16], [17].

These methods can be easy to extend to IPT systems [18]–[23]. However, in the IPT system, there are more resonant elements, the order of model is higher than resonant converter. Even the simplest IPT system (SS), is a 9th-order system [18]. Therefore, besides the model derivation, the model simplification is another challenging issue. Numerical method to reduce the system’s order is a simple way [21], [22]. In [21], the 11th-order system is reduced to a 2nd-order system by numerical calculation. However, this way requires large calculation and loses the physical insight. In [24], the author reduces the order of resonant converter by simplification of resonant capacitors. This method can also extend to IPT systems.

This paper is devoted to the small-signal model simplification for high-order IPT systems. The LCC-C compensated IPT system serves as an example to discuss the small signal model. In this paper, a small signal model from input to output is derived based on extended describing function. This method uses the fundamental harmonic approximation, and is popular for modeling. In order to gain more physical insight, the equivalent circuit of small signal model for elements in the IPT system is derived. The model for whole system is 13 order without any simplification. This model is too difficult to analyses. Based on the method proposed in [24], the order of small-signal circuit model is reduced by well dealing with the series and parallel resonance. The origin 13th-order system is simplified to a 7th-order one. The equivalent circuit model is derived for this IPT system. Based on this model the input-output gain, input impedance and output impedance will be derived. Finally, simulations are carried out for verification.

II. EQUIVALENT CIRCUIT MODEL

A. Static-State Characteristics

An unregulated IPT system works as a dc transformer (DCX) as shown in Fig. 1. This IPT stage consists of an inverter, LCC-C compensation network and a rectifier. \( L_1 \), \( C_1 \), and \( C_p \) forms the transmitting compensation (TX), and \( C_s \) is serially connected to the receiving (RX) coil. The coupler is
represents its T-model with three inductors. This LCC-C compensation can easily achieve load-independent output voltage under the following resonance equations:

\[
\begin{align*}
\frac{j\omega L_1}{j\omega C_1} - \frac{1}{j\omega C_1} &= j\omega L_p + \frac{1}{j\omega C_p} \\
\frac{1}{j\omega L_s} + \frac{1}{j\omega C_s} &= 0
\end{align*}
\]

In (1), \( \omega_s \) is the angular switching frequency. Under this condition, the output voltage gain \( G_{dc} \) is:

\[
G_{dc} = \frac{v_o}{v_{in}} = \frac{M}{L_1}.
\] (2)

In [10], a front-end stage controls \( v_{in} \) to regulate \( v_o \). Therefore, the small signal transfer function from input and output should be considered and is selected as an example to demonstrate the order-reduction method. The whole system consists of the inverter, the resonant tank, and the rectifier. The first step is to model each part by the EDF method.

**B. Equivalent Circuit Model of Inverter and Rectifier**

For the inverter, the input is approximated as a dc component, while the output using fundamental harmonic approximation as below:

\[
\begin{align*}
V_{AB} &\approx V_{AB} \sin \omega_s t \\
i_{AB} &\approx I_{AB} \sin \omega_s t
\end{align*}
\] (3)

Where \( V_{AB} \) and \( I_{AB} \) mean the magnitude of the voltage and current. \( V_{AB} \) and \( I_{AB} \) can be derived by Fourier decomposition:

\[
\begin{align*}
V_{AB} &= \frac{4}{\pi} \int_0^{\pi/2} v_{in} \cos \omega_s t dt \\
i_{AB} &= \frac{4}{\pi} \int_0^{\pi/2} i_{in} \cos \omega_s t dt
\end{align*}
\] (4)

And then \( V_{AB} \) is decomposed as:

\[
V_{AB} = \bar{V}_{AB} + \hat{V}_{AB},
\] (5)

where \( \bar{V}_{AB} \) means the average of \( V_{AB} \), and \( \hat{V}_{AB} \) is the small-signal perturbation. Similarly, it also has:

\[
\begin{align*}
\bar{V}_{in} &= \bar{v}_{in} + \hat{v}_{in} \\
\bar{i}_{AB} &= \bar{i}_{AB} + \hat{i}_{AB} \\
i_{in} &= \bar{i}_{in} + \hat{i}_{in}
\end{align*}
\] (6)

Based on (3)-(6), the small-signal voltage gain is easily derived as:

\[
\hat{v}_{AB} = 4 \hat{v}_{in}.
\] (7)

So, the small signal model for the inverter is shown in Fig.2.

**The modeling concept for the rectifier is the same as the inverter.** The voltage for input of rectifier \( v_{CD} \) can be approximated the fundamental harmonic, and the output is approximated as DC component. Due to the phase difference \( \phi \) between \( v_{AB} \) and \( v_{CD} \), the voltage and current of rectifier have to be separated to the sine part and cosine part:

\[
\begin{align*}
v_{CD} &= V_{CD,s} \sin \omega s t + V_{CD,c} \cos \omega s t \\
i &= I_{CD,s} \sin \omega s t + I_{CD,c} \cos \omega s t
\end{align*}
\] (8)

The phase difference \( \phi \) is dependent on the current \( I_{CD} \): \( \tan \phi = \frac{I_{CD,c}}{I_{CD,s}} \). So, the orthogonal components in (8) can be derived below:

\[
\begin{align*}
V_{CD,s} &= \frac{2}{\pi} \int_{\phi}^{\pi-\phi} V_o \sin \omega s t dt \\
V_{CD,c} &= \frac{2}{\pi} \int_{\phi}^{\pi-\phi} V_o \cos \omega s t dt
\end{align*}
\] (9)

Since the rectifier output current \( i_r = |i_{CD}| \), its magnitude is calculated by

\[
i_r = \frac{1}{2\pi} \int_0^{2\pi} |I_{CD} \sin \omega s t| dt = \frac{2}{\pi} \sqrt{I_{CD,s}^2 + I_{CD,c}^2}.
\] (10)

Considering the perturbations, all the above state variables are defined as:

\[
\begin{align*}
I_{CD,s} &= \bar{I}_{CD,s} + \hat{I}_{CD,s} \\
I_{CD,c} &= \bar{I}_{CD,c} + \hat{I}_{CD,c} \\
V_{CD,s} &= \bar{V}_{CD,s} + \hat{V}_{CD,s} \\
V_{CD,c} &= \bar{V}_{CD,c} + \hat{V}_{CD,c}
\end{align*}
\] (11)

The small signal model can be derived by First order Taylor expansion for (9) and (10):

\[
\begin{align*}
\hat{V}_{CD,s} &= R_s \hat{I}_{CD,s} + H_s \hat{I}_{CD,c} + 2H_s \hat{v}_{o} \\
\hat{V}_{CD,c} &= H_r \hat{I}_{CD,s} + R_s \hat{I}_{CD,c} + 2H_r \hat{v}_{o} \\
i_r &= H_s \hat{I}_{CD,s} + H_c \hat{I}_{CD,c}
\end{align*}
\] (12)

The detailed derivation is given in [15], and the equivalent model is shown in Fig.3. This small signal model has both sine part and cosine part.
C. Equivalent Circuit Model of the Resonant Tank

The resonant tank includes the coupling coils and compensation networks. In this paper, T-model is used to represent coupling coils by three separate inductors, and then the model only needs to deal with inductors and capacitors. For any inductor current ($i_L$) or capacitor voltage ($v_C$), assume $i_L(t) = I_L e^{j\omega t}, v_C(t) = V_C e^{j\omega t}$, $I_L = \hat{i}_L$, and $V_C = \hat{V}_C + v_C$, and then the general small signal model are derived as

\[
\begin{align*}
\hat{v}_L &= L \frac{di_L}{dt} + j\omega L \hat{i}_L, \\
\hat{i}_C &= C \frac{dv_C}{dt} + j\omega C \hat{v}_C.
\end{align*}
\] (13)

The small signal model for inductor and capacitor is given in Fig. 4. There is a complex term in the original circuit, i.e., $j\omega L$, which fails the circuit simulation. Meanwhile, the model of the rectifier also includes two orthogonal parts and cannot be directly connected to the resonant tank. So, the model are separated to sine and cosine part, i.e., $\hat{i}_L = \hat{i}_{L,s} + j\hat{i}_{L,c}$ and $\hat{v}_C = \hat{v}_{C,s} + j\hat{v}_{C,c}$. Based on this decomposition (13) is written as (14). So, the model is separated into orthogonal parts in Fig. 4:

\[
\begin{align*}
\hat{v}_{L,s} &= L \frac{di_{L,s}}{dt} - \omega_s L \hat{i}_{L,s}, \\
\hat{v}_{L,c} &= L \frac{di_{L,c}}{dt} + \omega_s L \hat{i}_{L,c}, \\
\hat{i}_{C,s} &= C \frac{dv_{C,s}}{dt} - \omega_s C \hat{v}_{C,s}, \\
\hat{i}_{C,c} &= C \frac{dv_{C,c}}{dt} + \omega_s C \hat{v}_{C,c}.
\end{align*}
\] (14)

III. Reduced-Order System Model

Based on the derivation above, the original small-signal model of the LCC-C compensated system can be obtained, which is a 13th-order circuit. This system is complex and hard to analyze. Therefore, a proper order-reduction method is preferred to simplify the model.

In this paper, a general method is proposed to reduce the order. According to [24], an approximation can be used to simplify the resonant capacitor branch in resonant tank. When the perturbation frequency ($s$) is much smaller than the switching frequency $\omega_s$, the capacitor in parallel with the complex impedance is simplified as an equivalent series branch. The mathematical analysis is shown in (15). From the circuit point of view, the proposed approximation is shown in Fig. 5.

\[
\frac{\hat{v}_C}{\hat{i}_c} = \frac{1}{sC + j\omega_s C} = \frac{1 + j\frac{s}{\omega_s}}{j\omega_s C (1 + \frac{s^2}{\omega_s^2})},
\] (15)

By separating the model of each resonant components to sine and cosine part, the final model is generated as shown in Fig. 7. In the resonant tank, there are 6 independent loops, so the order for this resonant tank is 6. Combining with the rectifier, the final model is a 7th-order system. Based on this simplified equivalent circuit, the original 13th-order system is reduced to a 7th-order one. By using Kirchhoff’s law, the state variable in Fig. 7 can be expressed as: $Ax = y$. The parameter of equations are shown in (16) - (19). By solving these equations, transfer function from input-output: $G_{ac}(s) = \frac{\hat{v}_o}{\hat{v}_{in}}$ can be derive. This model can also be used to derive the input impedance $Z_{in}(s)$ and output impedance $Z_{out}(s)$.
In this section, the simulation for small signal model is obtained by SIMPLIS to verify the model accuracy. The circuit parameters are shown in Tab. I. The input-to-output transfer function is measured in the simulation as shown in Fig. 8 [i.e., the blue curve]. The red dot line gives the result using reduced-order model. This model is accurate up to 1/3 switching frequency.

The comparison between simulation and model for input impedance $Z_{in}(s)$ is shown in Fig. 9, both magnitude and phase are accurate up to 1/5 switch frequency.

The output impedance $Z_{out}(s)$ for this IPT system is also verified. The comparison between simulation and model is shown in Fig. 10, the magnitude of the output impedance is accurate almost at any frequency, while the phase of output impedance is only accurate at high frequency.

The results from Fig. 8 to Fig. 10 show that the accuracy of this reduced model is high enough to build the front-end controller.
\[
A = \begin{bmatrix}
\begin{array}{cccccc}
  s(L_1 + 1/\omega_s^2 C_1) & -L_1 \omega_s & -s/\omega_s C_1 & -1/\omega_s C_1 & 0 & 0 & 0 \\
  L_1 \omega_s - 1/\omega_s^2 C_1 & s(L_1 + 1/\omega_s^2 C_1) & 1/\omega_s C_1 & -1/\omega_s C_1 & 0 & 0 & 0 \\
  sL_1 & -sL_1 & s(L_p' - M) & -X_{eq1} - \omega_s M & -sM & -sM & 0 \\
  \omega_s L_1 & sL_1 & X_{eq1} + \omega_s M & s(L_p' - M) & \omega_s M & -sM & 0 \\
  0 & 0 & 0 & -sM & \omega_s M & -sM & 0 \\
  0 & 0 & \omega_s M & -sM & -X_{eq2s} - \omega_s M & s(L_s' + M) + R_c & 2H_c \\
  0 & 0 & 0 & 0 & -H_s & -H_c & \frac{1}{\pi} + C_s
\end{array}
\end{bmatrix}
\]

\(x = \begin{bmatrix} \hat{i}_{1,s}, \hat{i}_{1,c}, \hat{i}_{p,s}, \hat{i}_{p,c}, \hat{i}_{2,s}, \hat{i}_{2,c}, \hat{v}_o \end{bmatrix}^T\)  \\
\(y = \begin{bmatrix} \frac{1}{2} \hat{v}_{in} & 0 & \frac{1}{2} \hat{v}_{in} & 0 & 0 & 0 & 0 \end{bmatrix}^T\)

(16)  
(17)  
(18)

\(L_p' = L_p - M + \frac{1}{\omega_s^2 C_p}; L_s' = L_s - M + \frac{1}{\omega_s^2 C_s}; X_{eq1} = \omega_s L_p - \frac{1}{\omega_s C_p} - \omega_s M\)  

\(X_{eq2s} = \omega_s L_2 - \frac{1}{\omega_s C_2} - \omega_s M - H_{rs}; X_{eq2c} = \omega_c L_2 - \frac{1}{\omega_c C_2} - \omega_s M + H_{rc}\)

(19)

\[x = [\hat{i}_{1,s}, \hat{i}_{1,c}, \hat{i}_{p,s}, \hat{i}_{p,c}, \hat{i}_{2,s}, \hat{i}_{2,c}, \hat{v}_o]^T\]

\[y = \left[ \frac{1}{2} \hat{v}_{in}, 0, \frac{1}{2} \hat{v}_{in}, 0, 0, 0, 0 \right]^T\]

Fig. 9. Input impedance \((Z_{in}(s))\) comparison between model-based calculation and SIMPLIS-based simulation.

Fig. 10. Output impedance \((Z_{out}(s))\) comparison between model-based calculation and SIMPLIS-based simulation.

V. CONCLUSION

In this paper, the small signal model for LCC-C IPT system is derived based on the extended describing function. A general method is proposed to reduce the order of system. The example LCC-C IPT system with 13 orders is simplified to a 7th-order system. The proposed method has more physical insight and maintains high accuracy. It is also suitable for any other IPT systems and resonant converters.

REFERENCES


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