Digital Control For Power Supply Engineers

Robert V. White

Embedded Power Labs
Highlands Ranch, Colorado, USA

www.embeddedpowerlabs.com

Seminar Overview

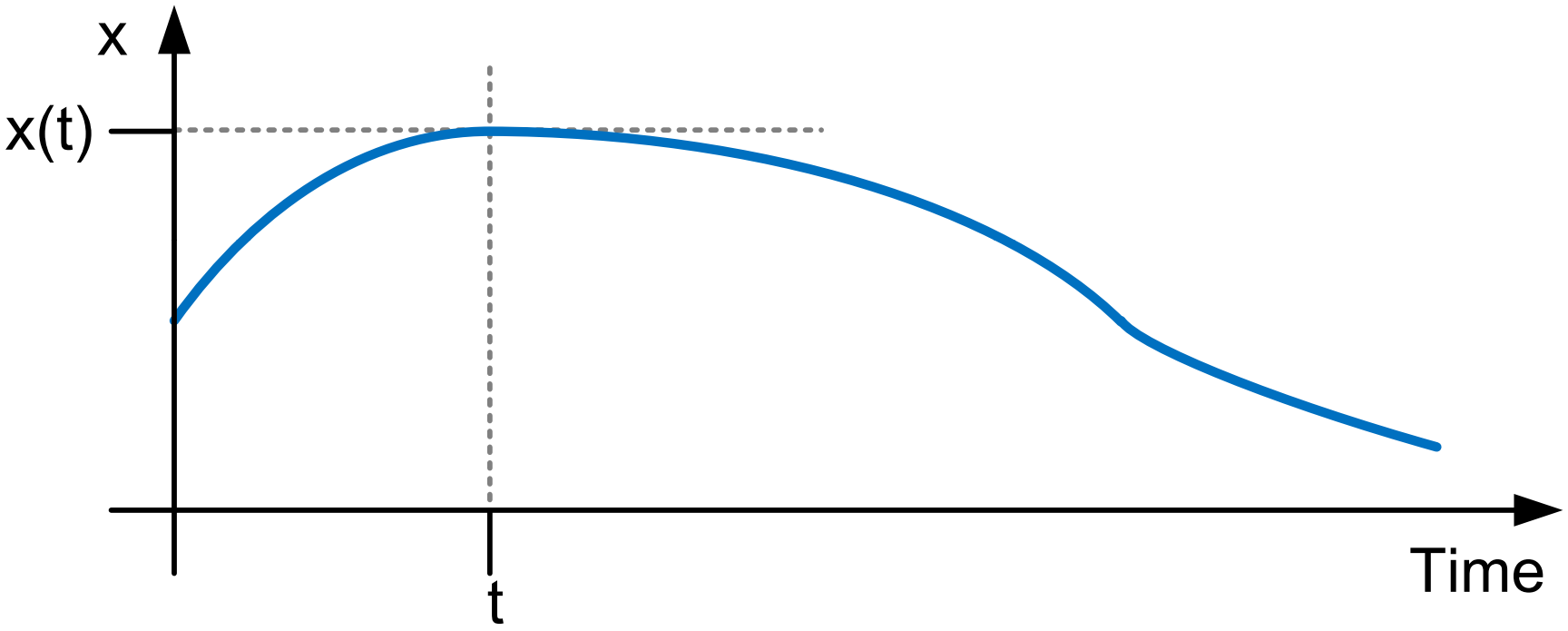
- Back to School
  - Discrete Time Concepts
  - The *z* Transform & Transfer Functions
  - Continuous ⇔ Discrete Time Conversion
- Digital Control Of Power Supplies
  - Sampling
  - Compensator
  - Digital PWM
- Benefits Enabled By Digital Control
- A Look Ahead

IP Disclaimer

- There Are A Lot Of Patents On Digital Control For Power Converters
- I Do Not Offer Any Assurance That Techniques Discussed In This Seminar Do Not Infringe Someone's Patent
- If You Have Questions About Whether A Technique You Are Considering Using Infringes Someone's Patent, Talk To The Attorney Of Your Choice

Discrete Time Foundation
"Back To School"

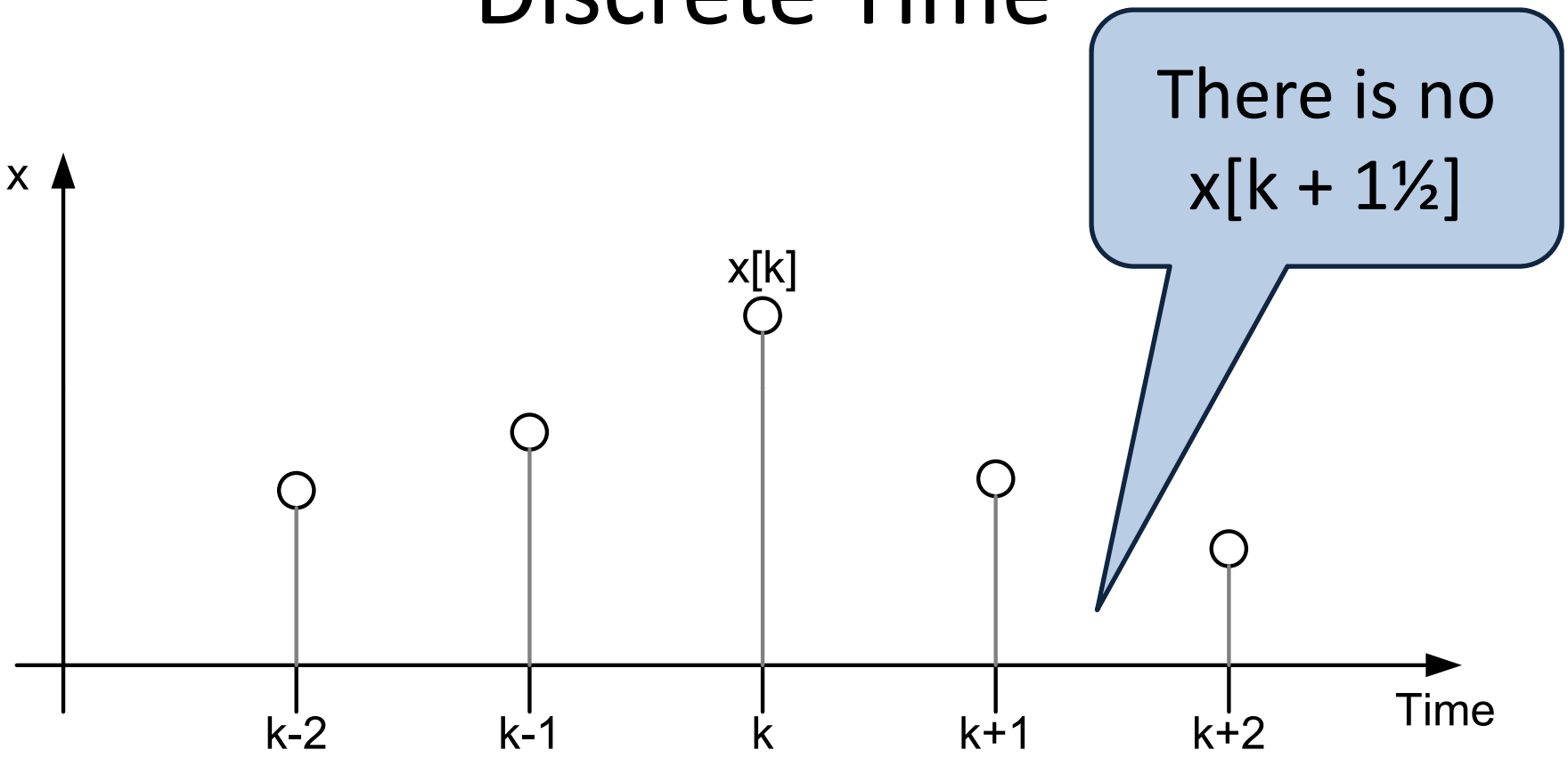
Continuous Time



For every t there is an x(t)

Example: $x(t) = X_0 \cdot \sin(\omega \cdot t)$

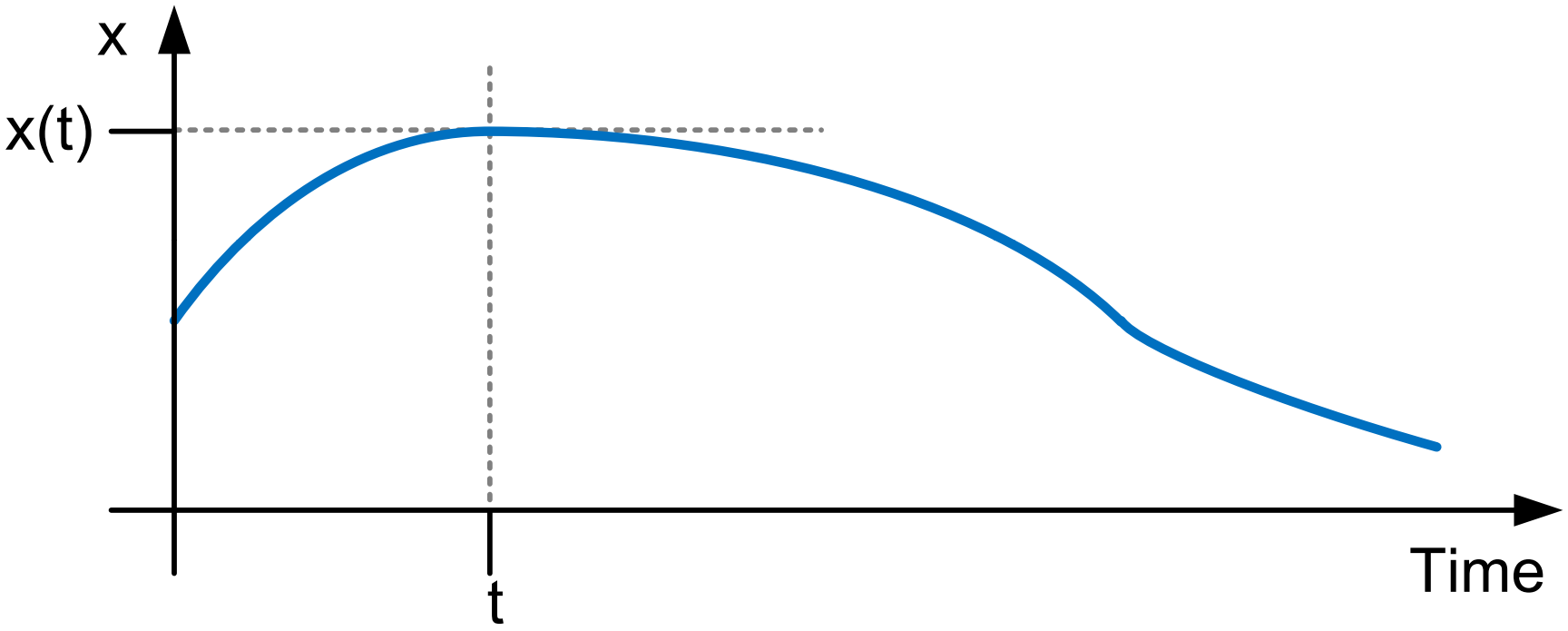
Discrete Time



x[k] is a sequence of values

Example: $x[k] = X_0 \cdot \sin(k)$, k is an integer

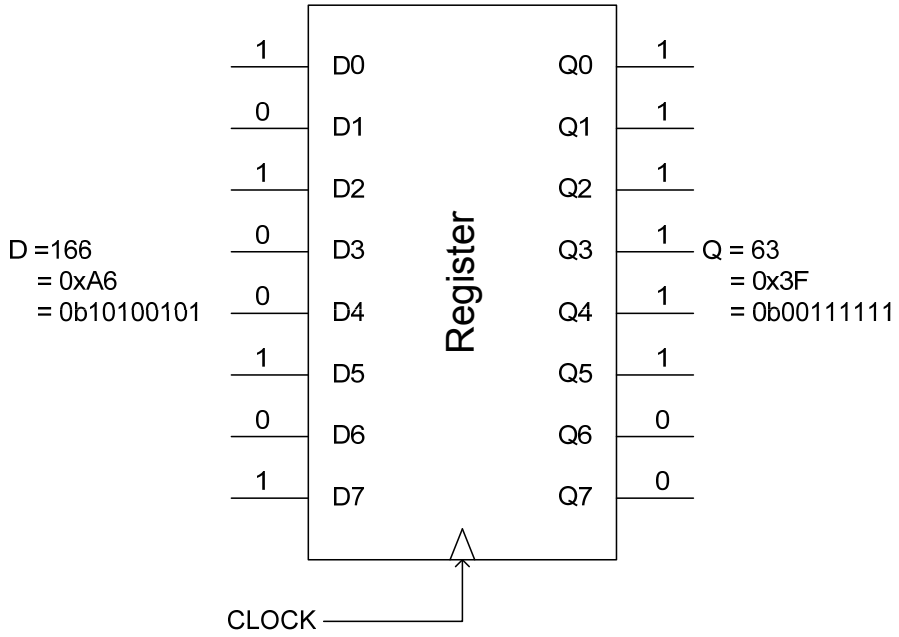
Analog Value



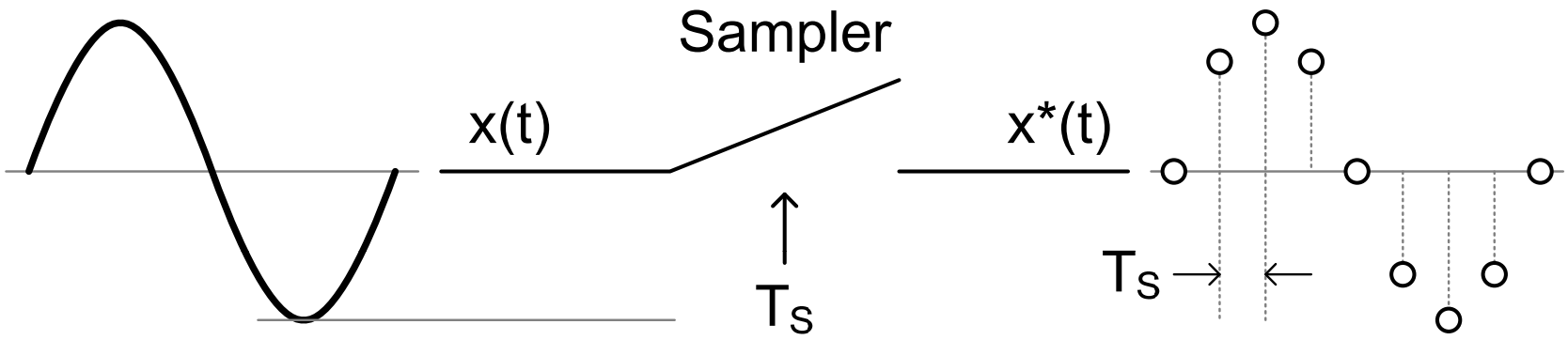
x(t) can be any real number

Digital Value

- A Digital Value is a value represented in a digital circuit
- Digital values are discrete, or quantized



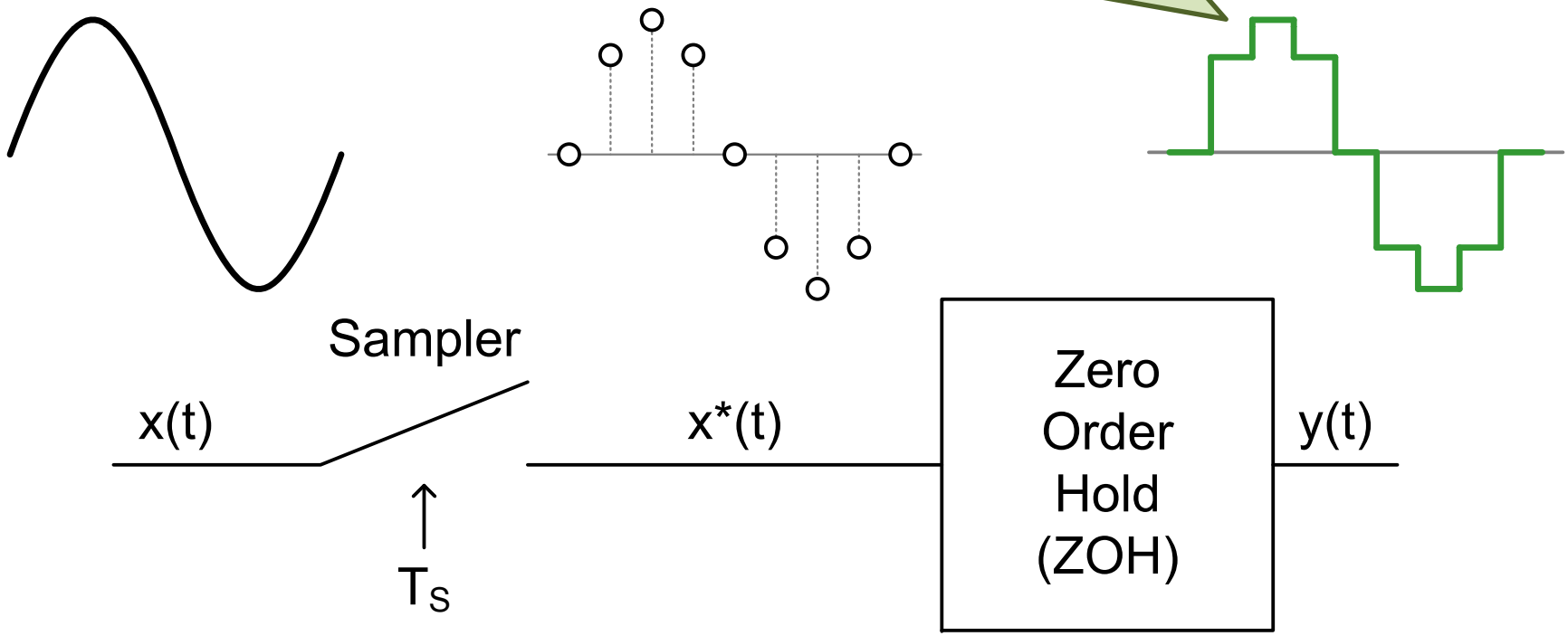
Continuous To Discrete Time Conversion



- Sampling: Process to convert a continuous time signal x(t) to a discrete time signal x*(t)
- Sampling interval: T_S or T_{SAMPLE}
- Sampling frequency: F_S or $F_{SAMPLE} = 1/T_S$

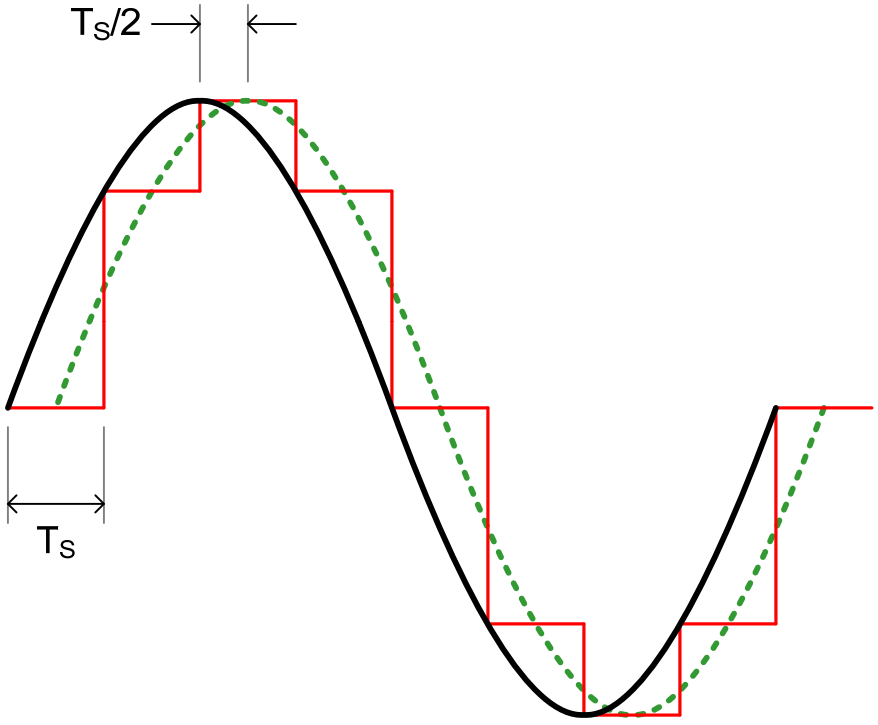
Sampling Plus ZOH

ZOH keeps output at same value until a new value appears at the input



Sampling Delay

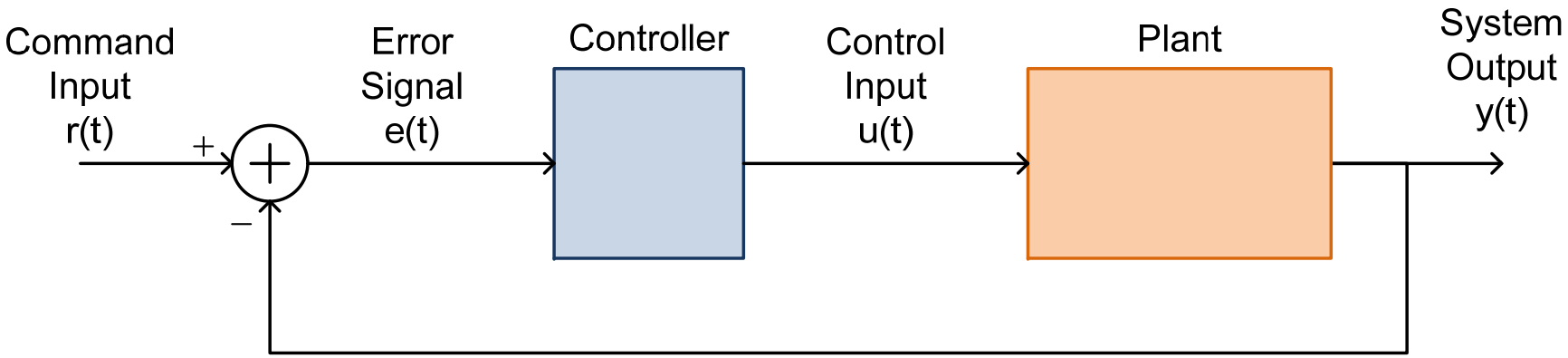
- ZOH output signal is delayed, on average, by one half the sampling interval
- This leads to additional phase shift in control loops



$$\phi(f) = \frac{f}{2 \cdot F_S} \cdot 360° = \frac{f}{F_S} \cdot 180°$$

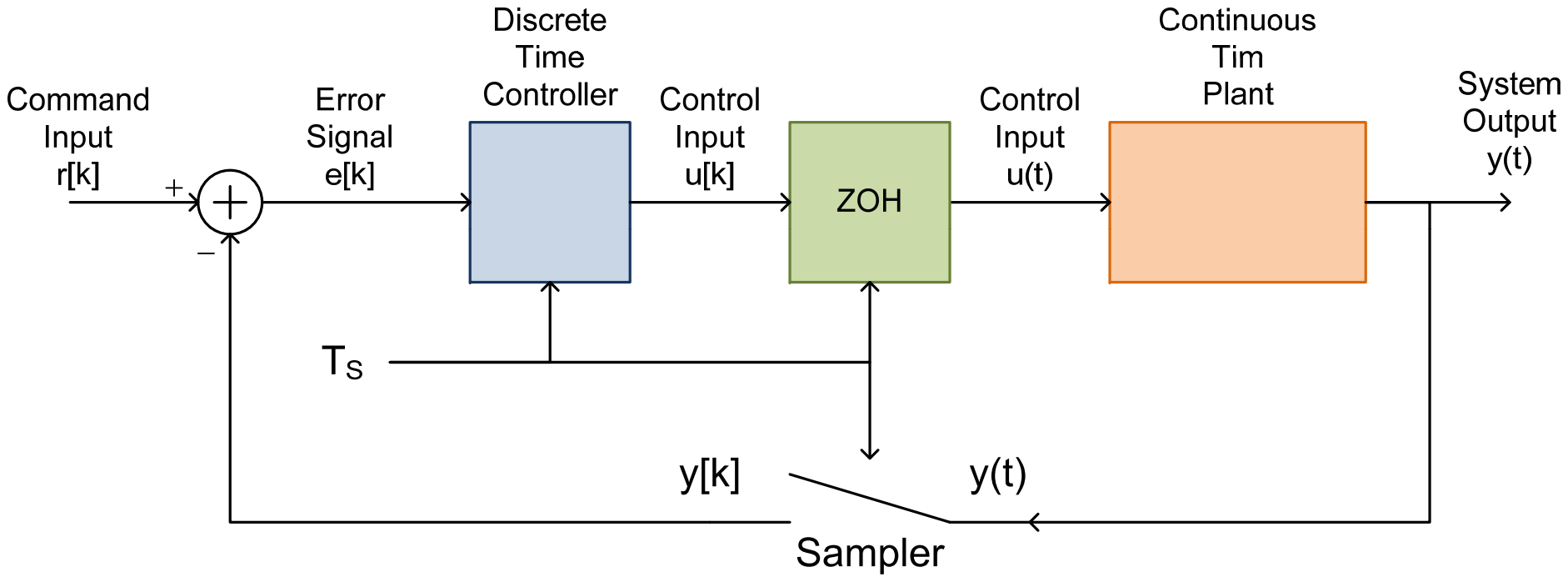
$$\phi\left(\frac{F_S}{10}\right) = \frac{\left(\frac{F_S}{10}\right)}{F_S} \cdot 180° = 18°$$

Continuous Time Control Loop



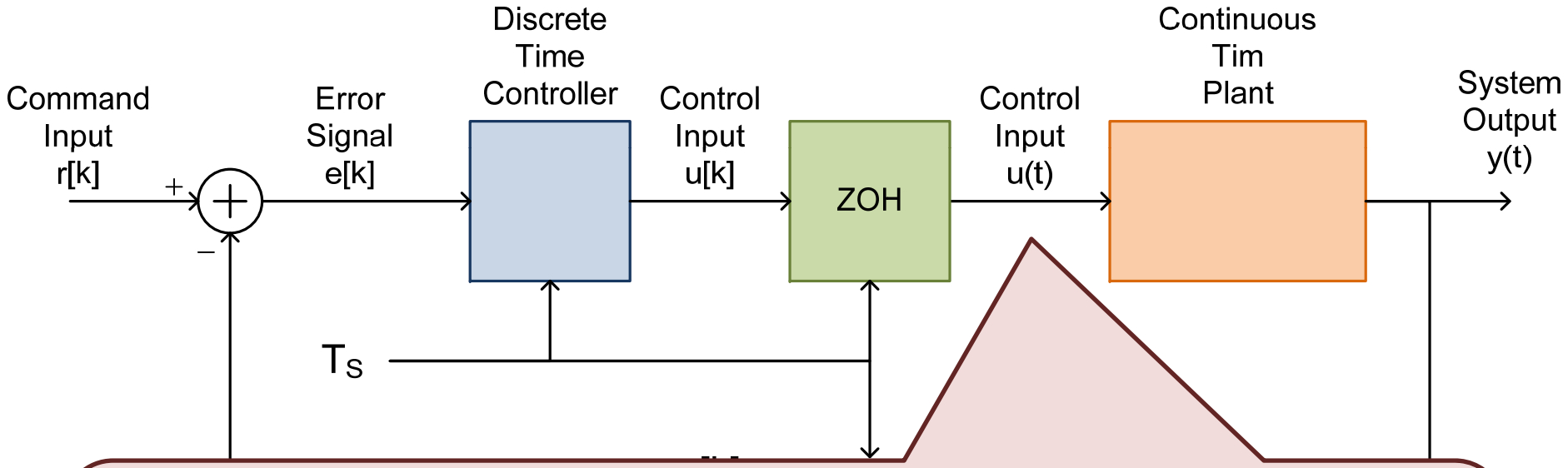
How do we convert to a discrete time controller?

Continuous Time Plant With Discrete Time Controller



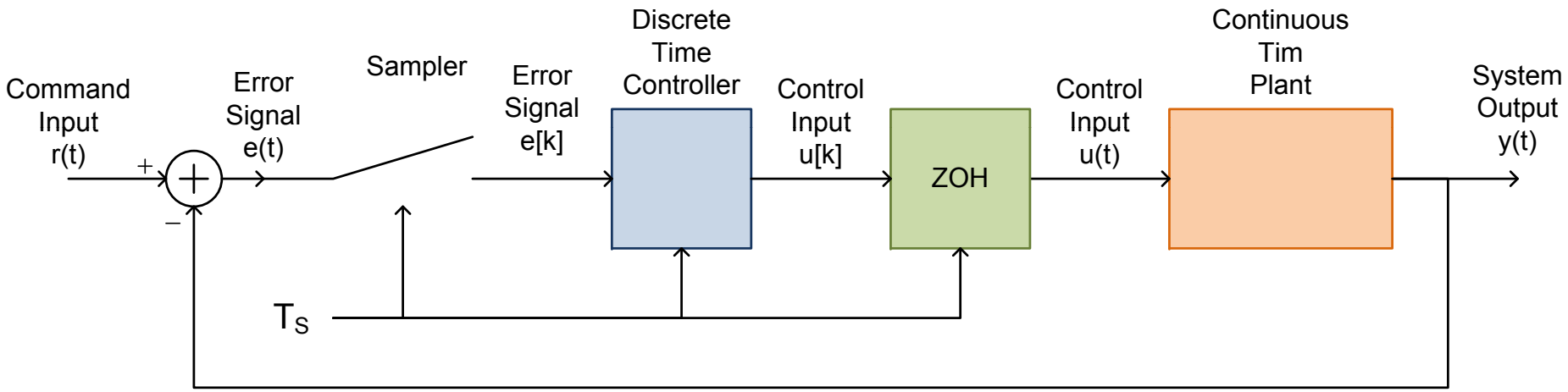
Systems with continuous time and discrete time elements are “sampled data systems”

Continuous Time Plant With Discrete Time Controller

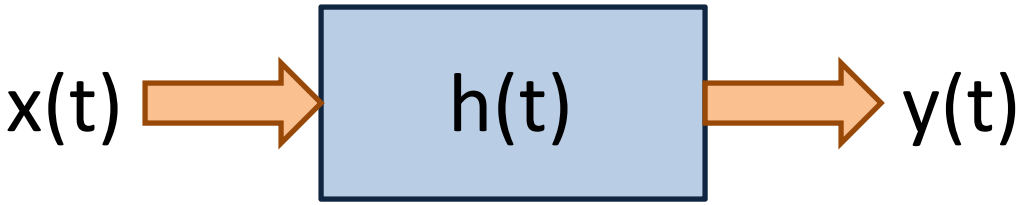


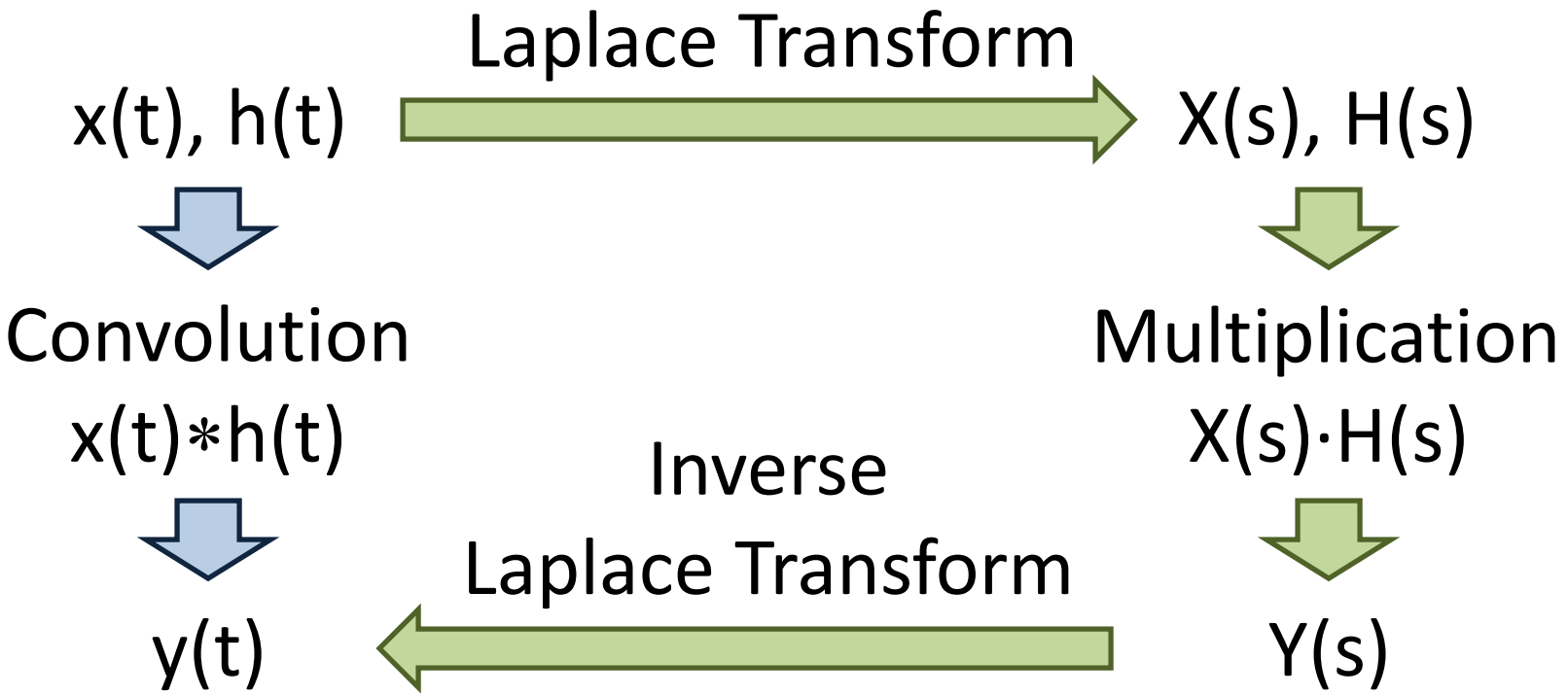
If the sampling rate is much faster than the plant dynamics, the control looks like continuous time to the plant

Continuous Time Plant With Discrete Time Controller

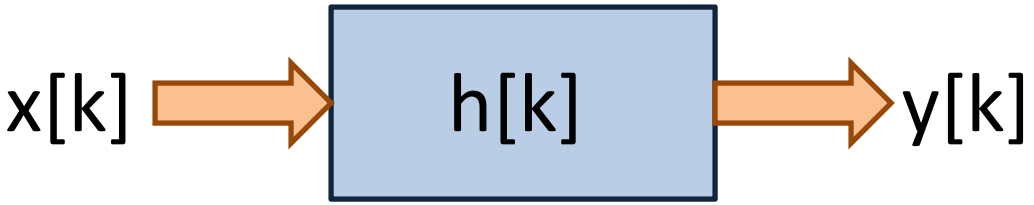


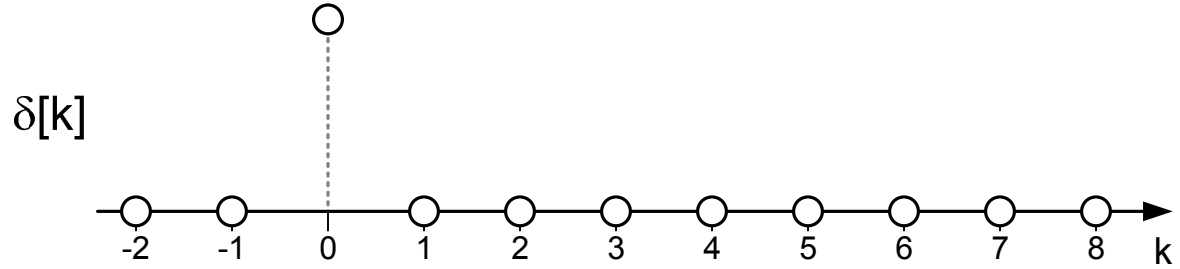
Solving Continuous Time Systems

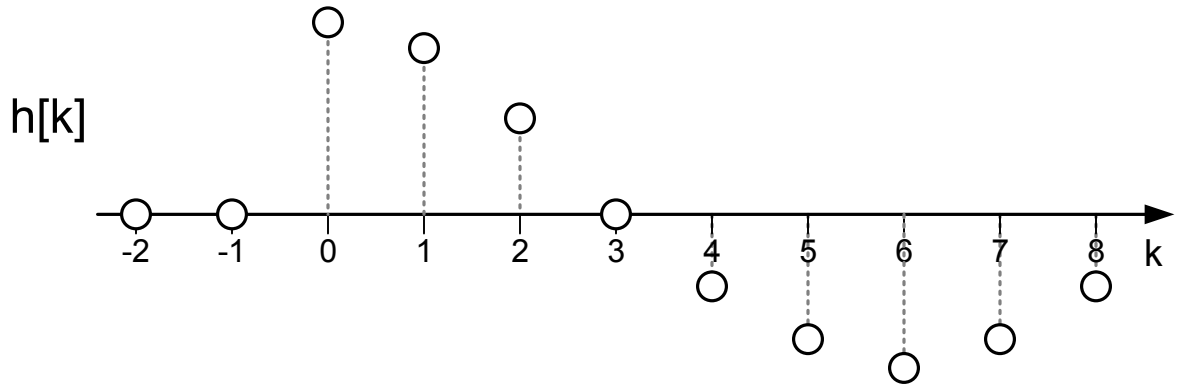




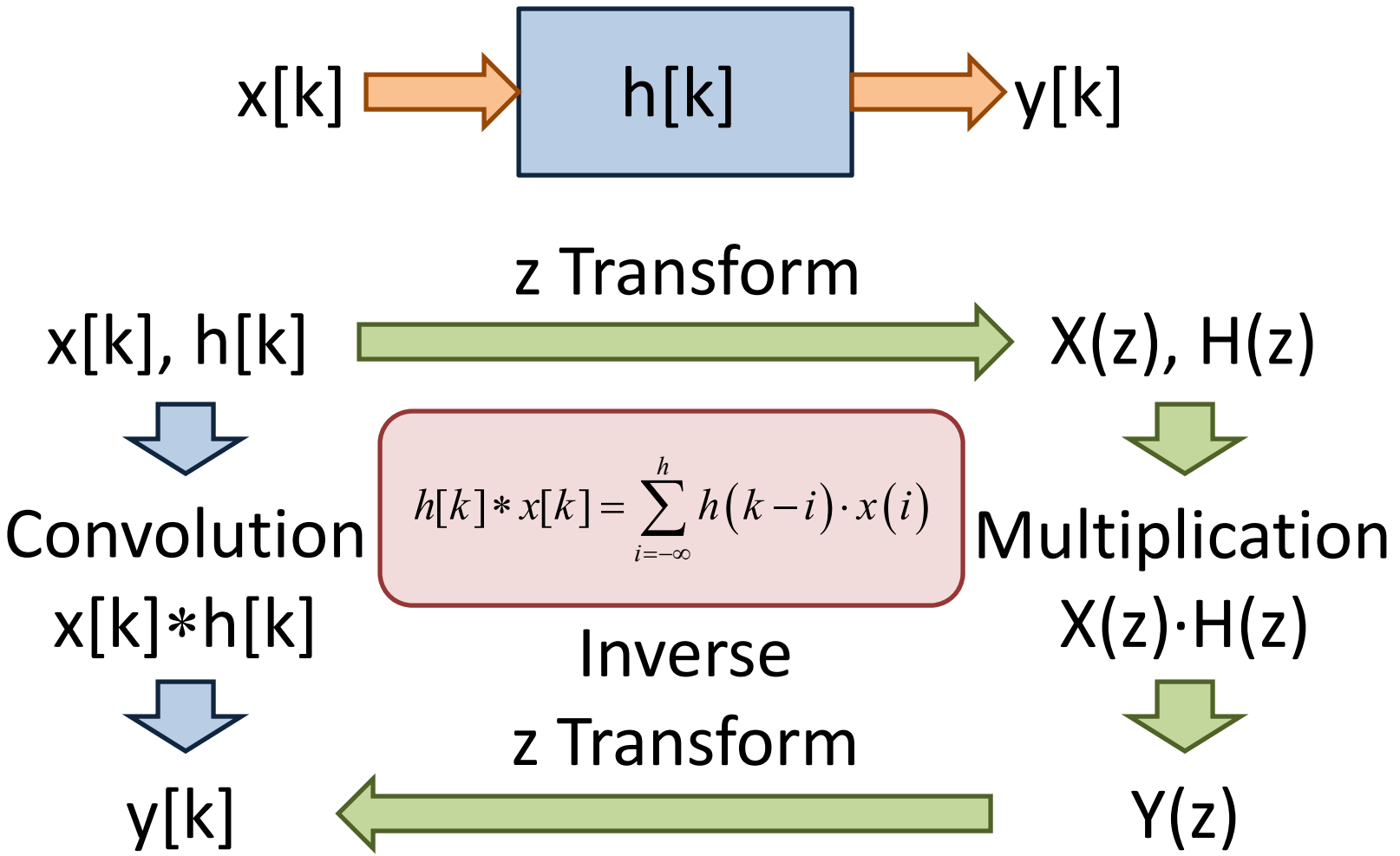
Solving Discrete Time Systems







Solving Discrete Time Systems



The *z* Transform

Laplace Transform

$$\mathcal{L}\{x(t)\} = X(s) = \int_0^{\infty} x(t) \cdot e^{-st} \cdot dt$$

$$s = \sigma + j \cdot \omega$$

z Transform

$$\mathcal{Z}\{x[k]\} = X(z) = \sum_{k=0}^{\infty} x[k] \cdot z^{-k}$$

z is a complex number

z Transform Examples

Impulse

$$x[k] = \delta[k]$$

$$X(z) = \sum_{k=0}^{\infty} \delta[k] \cdot z^{-k}$$

$$= 1 \cdot z^{-0}$$

$$= 1$$

Step

$$x[k] = 1[k] = \begin{cases} 0, k < 0 \\ 1, k \geq 0 \end{cases}$$

$$X(z) = \sum_{k=0}^{\infty} 1[k] \cdot z^{-k}$$

$$= \sum_{k=0}^{\infty} z^{-k}$$

$$= \frac{1}{1 - z^{-1}}$$

$$= \frac{z}{z - 1}$$

Exponential

$$x[k] = e^{-a \cdot k \cdot T_S} \cdot 1(t)$$

$$X(z) = \sum_{k=0}^{\infty} e^{-a \cdot k \cdot T_S} \cdot z^{-k}$$

$$= \sum_{k=0}^{\infty} \left(e^{-a \cdot T_S} \cdot z^{-1} \right)^k$$

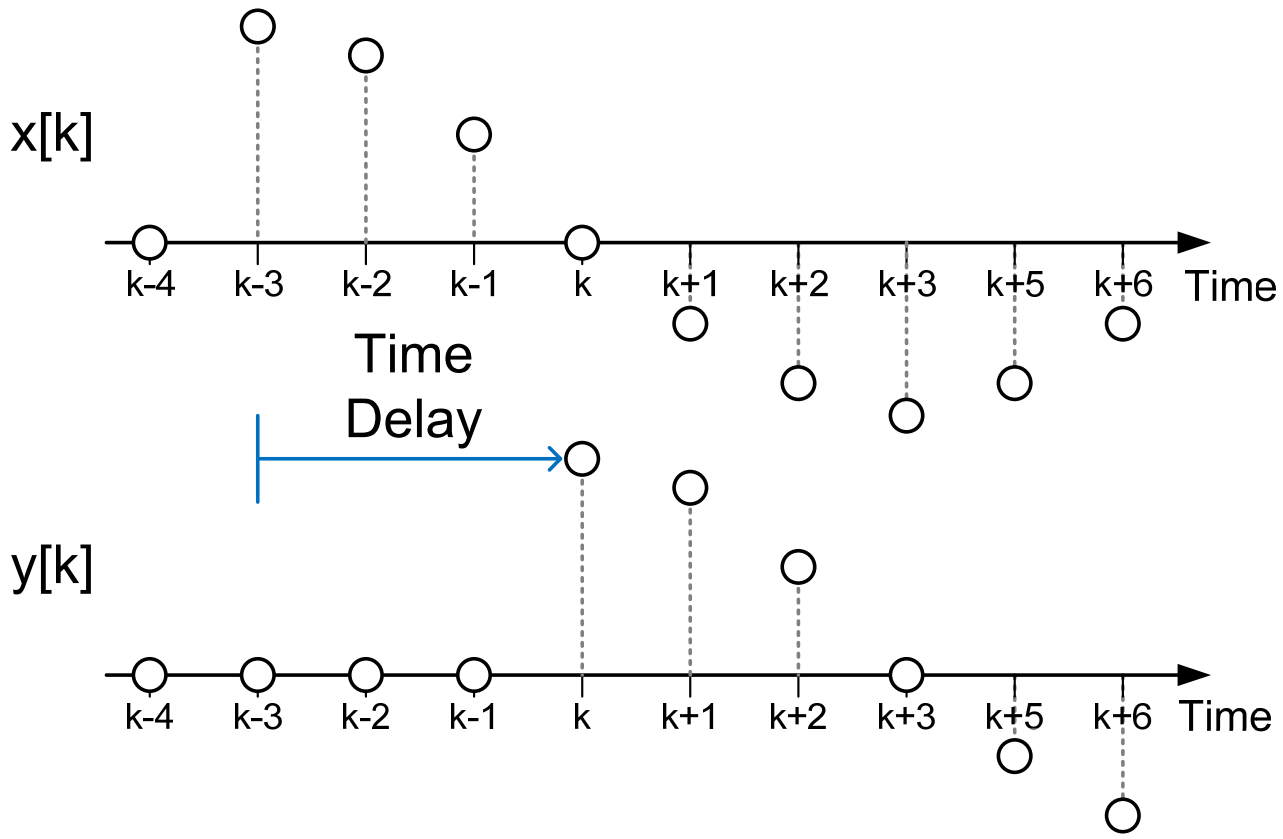
$$= \frac{1}{1 - e^{-a \cdot T_S} \cdot z^{-1}}$$

$$= \frac{z}{z - e^{-a \cdot T_S}}$$

$$|z| > e^{-a \cdot T_S}$$

EMBEDDED POWER LABS

z Transform Of a Delay



$$y[k] = x[k-3]$$

z Transform Of a Delay

- Arbitrary Delay
- Take z Transform
- Change Variables

$$y[k] = x[k-n]$$

$$Y(z) = \sum_{k=0}^{\infty} x[k-n] \cdot z^{-k}$$

$$k = l + n \Rightarrow l = k - n$$

$$Y(z) = \sum_{l=-n}^{\infty} x[l] \cdot z^{-l-n}$$

$$= z^{-n} \cdot \sum_{l=-n}^{\infty} x[l] \cdot z^{-l}$$

- Note That:

$$l < 0 \Rightarrow x[l] = 0$$

- Result

$$Y(z) = z^{-n} \cdot \sum_{l=0}^{\infty} x[l] \cdot z^{-l}$$

$$= z^{-n} \cdot X(z)$$

EMBEDDED POWER LABS

z Transform Properties

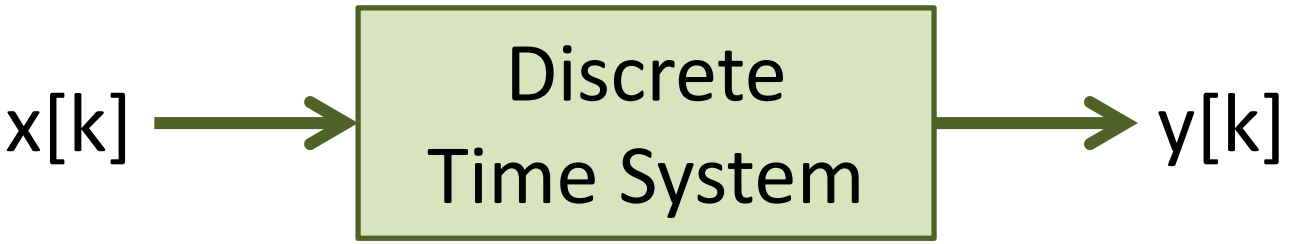
- Linearity

$$\mathcal{Z}\left(a \cdot x[k] + b \cdot y[k]\right) = a \cdot X(z) + b \cdot Y(z)$$

- Time Invariant (Stationary)

$$\mathcal{Z}\left(x[k-n]\right) = z^{-n} \cdot X(z)$$

Difference Equations

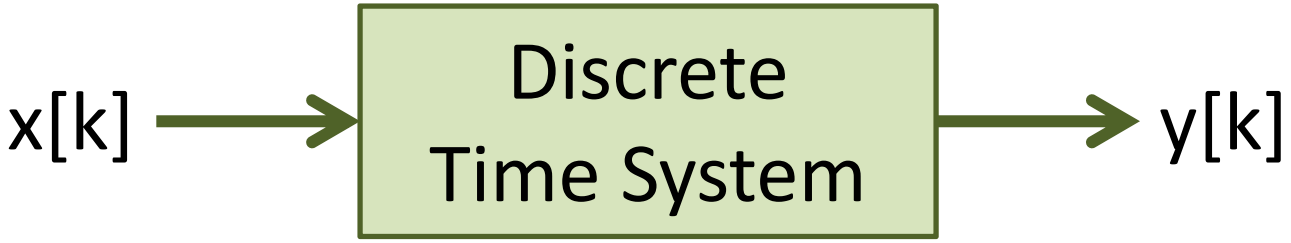


$$y_k = y[k] = f\left(y_{k-1}, y_{k-2}, \ldots, y_0, x_k, x_{k-1}, \ldots, x_0\right)$$

$$y_k = -a_1 \cdot y_{k-1} - a_2 \cdot y_{k-2} - \ldots - a_n \cdot y_{k-n} + b_0 \cdot x_k + b_1 \cdot x_{k-1} + b_2 \cdot x_{k-2} + \ldots + b_m \cdot x_{k-m}$$

Easily Computed

Discrete Time Transfer Function



$$y_k = -a_1 \cdot y_{k-1} - a_2 \cdot y_{k-2} - \ldots - a_n \cdot y_{k-n} + b_0 \cdot x_k + b_1 \cdot x_{k-1} + b_2 \cdot x_{k-2} + \ldots + b_m \cdot x_{k-m}$$

$$Y(z) = Z\left\{ \begin{matrix} -a_1 \cdot y_{k-1} - a_2 \cdot y_{k-2} - \ldots - a_n \cdot y_{k-n} \\ +b_0 \cdot x_k + b_1 \cdot x_{k-1} + b_2 \cdot x_{k-2} + \ldots + b_m \cdot x_{k-m} \end{matrix} \right\}$$

Discrete Time Transfer Function

$$Y(z) = \mathcal{Z}\{-a_1 \cdot y_{k-1} - a_2 \cdot y_{k-2} - \ldots - a_n \cdot y_{k-n}\}$$
$$+\mathcal{Z}\{+b_0 \cdot x_k + b_1 \cdot x_{k-1} + b_2 \cdot x_{k-2} + \ldots + b_m \cdot x_{k-m}\}$$

$$Y(z) = \mathcal{Z}\{-a_1 \cdot y_{k-1}\} + \mathcal{Z}\{-a_2 \cdot y_{k-2}\} + \ldots \mathcal{Z}\{-a_n \cdot y_{k-n}\}$$
$$+\mathcal{Z}\{b_0 \cdot x_k\} + \mathcal{Z}\{b_1 \cdot x_{k-1}\} + \ldots \mathcal{Z}\{b_m \cdot x_{k-m}\}$$

$$Y(z) = -a_1 \cdot \mathcal{Z}\{y_{k-1}\} - a_2 \cdot \mathcal{Z}\{y_{k-2}\} - \ldots - a_m \cdot \mathcal{Z}\{y_{k-m}\}$$
$$+b_0 \cdot \mathcal{Z}\{x_k\} + b_1 \cdot \mathcal{Z}\{x_{k-1}\} + \ldots + b_m \cdot \mathcal{Z}\{x_{k-m}\}$$

$$Y(z) = -a_1 \cdot z^{-1} \cdot Y(z) - a_2 \cdot z^{-2} \cdot Y(z) - \ldots - a_n \cdot z^{-n} \cdot Y(z)$$
$$+b_0 \cdot X(z) + b_1 \cdot z^{-1} \cdot X(z) + \ldots + b_m \cdot z^{-m} \cdot X(z)$$

EMBEDDED POWER LABS

Discrete Time Transfer Function

$$Y(z)+a_1\cdot z^{-1}\cdot Y(z)+a_2\cdot z^{-2}\cdot Y(z)+\ldots+a_n\cdot z^{-n}\cdot Y(z)$$
$$=b_0\cdot X(z)+b_1\cdot z^{-1}\cdot X(z)+\ldots+b_m\cdot z^{-m}\cdot X(z)$$

$$\left(1+a_1\cdot z^{-1}+a_2\cdot z^{-2}+\ldots+a_n\cdot z^{-n}\right)\cdot Y(z)$$
$$=\left(b_0+b_1\cdot z^{-1}+\ldots+b_m\cdot z^{-m}\right)\cdot X(z)$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{b_0+b_1\cdot z^{-1}+\ldots+b_m\cdot z^{-m}}{1+a_1\cdot z^{-1}+a_2\cdot z^{-2}+\ldots+a_n\cdot z^{-n}}$$

Discrete Time Integrator

Continuous Time Reminder

$$y(t) = \int_0^t x(\tau) \cdot d\tau$$

$$Y(s) = \frac{1}{s} \cdot X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s}$$

$$y[k] = \sum_0^k x[k]$$

$$= x[k] + \sum_0^{k-1} x[k]$$

$$= x[k] + y[k-1]$$

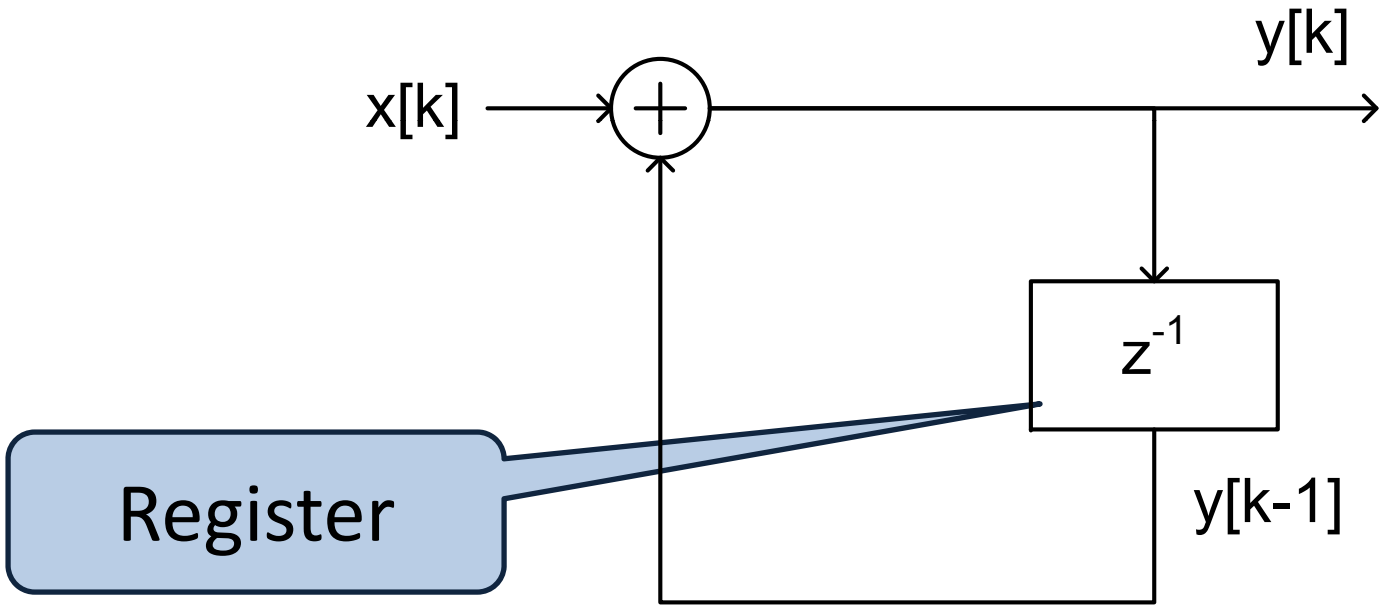
$$Y(z) = X(z) + z^{-1} \cdot Y(z)$$

$$\left(1 - z^{-1}\right) \cdot Y(z) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - z^{-1}}$$

Discrete Time Integrator

$$y[k] = x[k] + y[k-1]$$



Example: Discrete Time Pole

Transfer Function

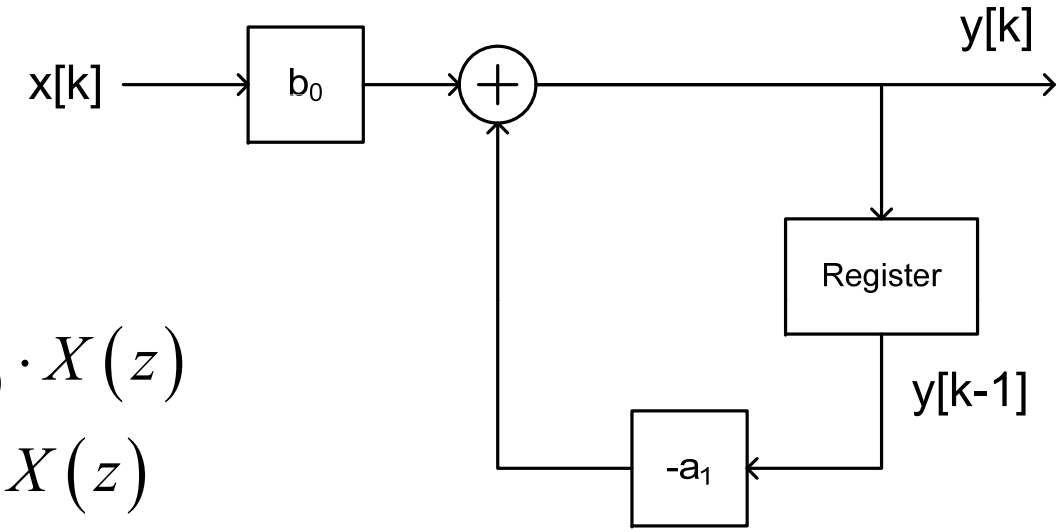
$$y_k = -a_1 \cdot y_{k-1} + b_0 \cdot x_k$$

$$Y(z) = -a_1 \cdot z^{-1} \cdot Y(z) + b_0 \cdot X(z)$$

$$Y(z) + a_1 \cdot z^{-1} \cdot Y(z) = b_0 \cdot X(z)$$

$$\left(1 + a_1 \cdot z^{-1}\right) \cdot Y(z) = b_0 \cdot X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0}{1 + a_1 \cdot z^{-1}}$$



Pole Frequency

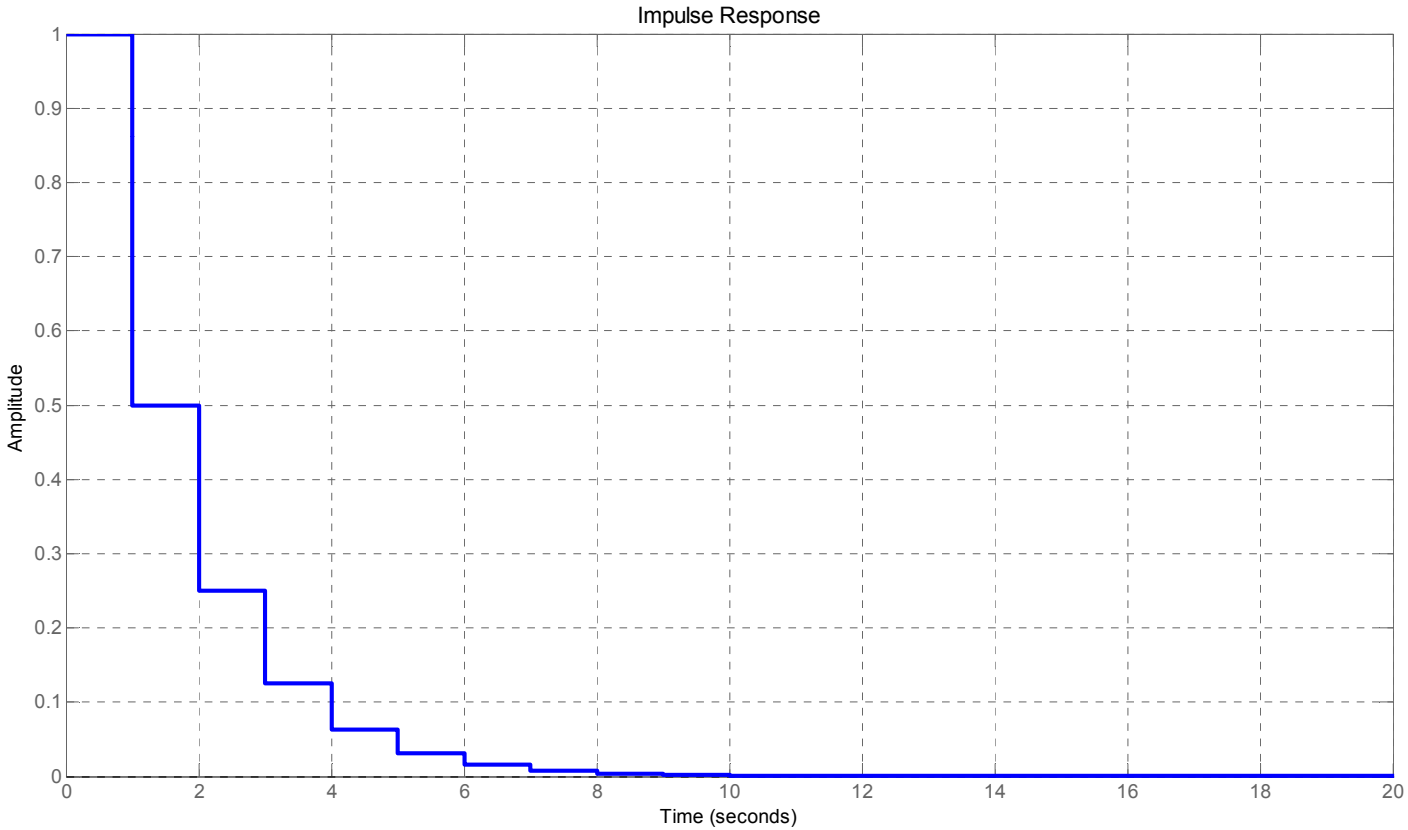
$$1 + a_1 \cdot z^{-1} = 0$$

$$-a_1 \cdot z^{-1} = 1$$

$$z = -a_1$$

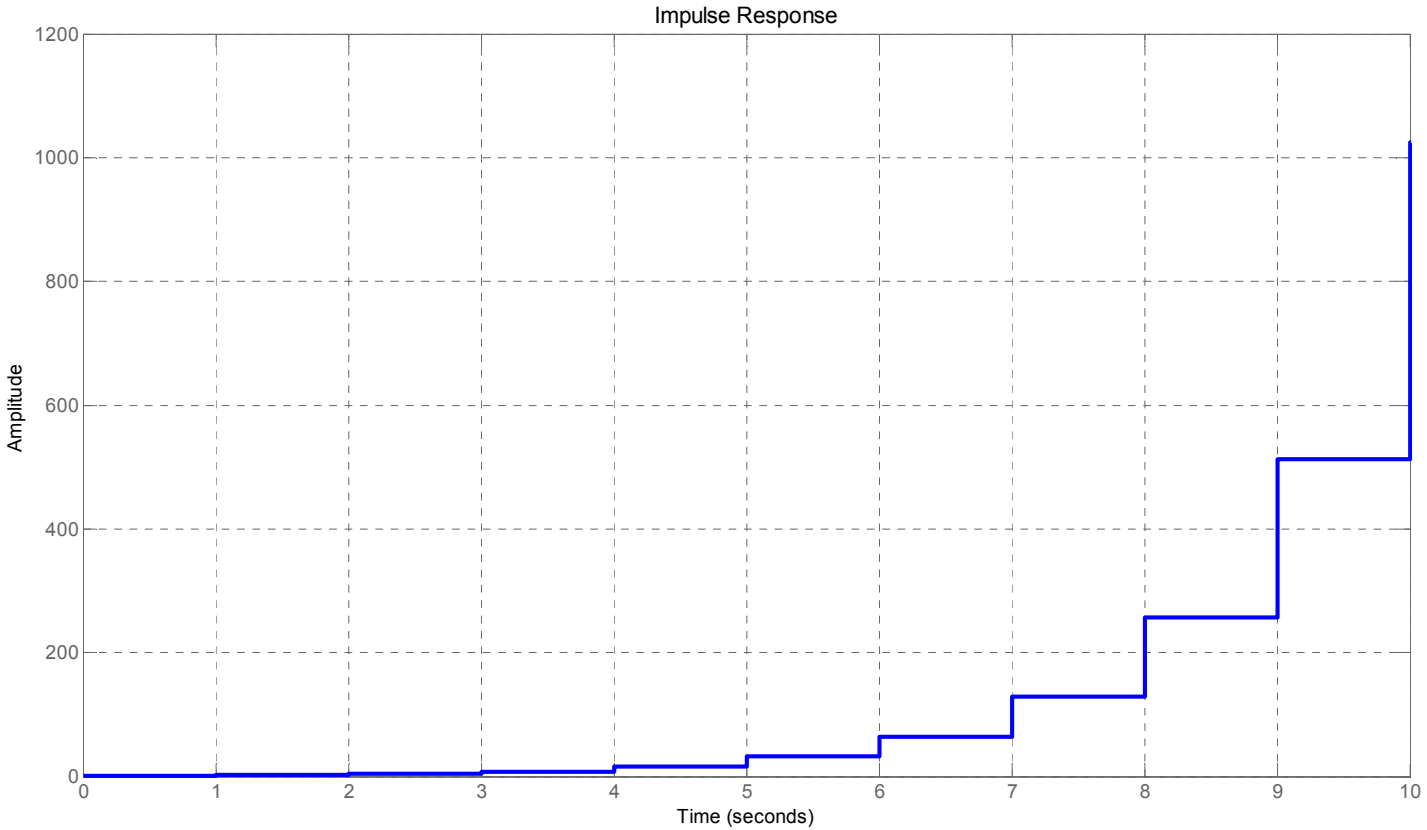
Example: Discrete Time Pole

$$H(z) = \frac{1}{1 + a \cdot z^{-1}} = \frac{z}{z + a} \qquad z = \frac{1}{2} \qquad a = -z = -\frac{1}{2}$$



Example: Discrete Time Pole

$$H(z) = \frac{1}{1 + a \cdot z^{-1}} = \frac{z}{z + a} \qquad z = 2 \qquad a = -2$$

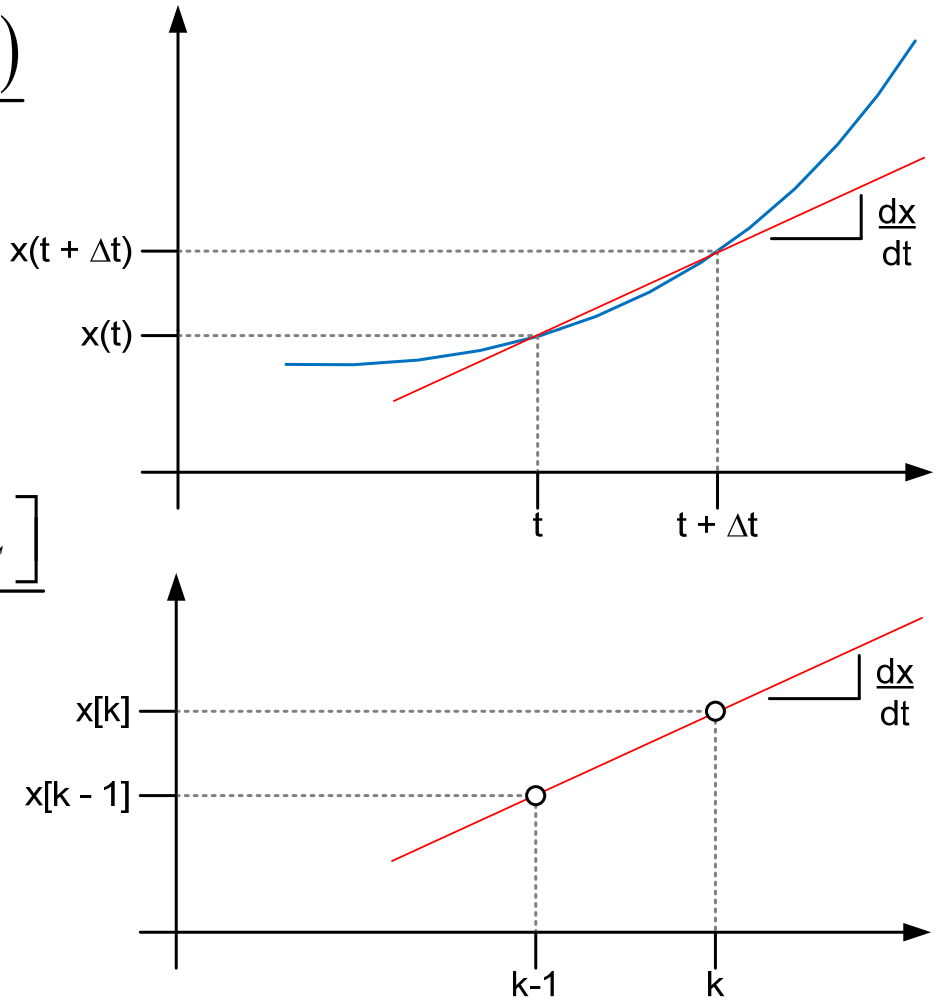


Discrete Time Derivative

$$y(t) = \frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

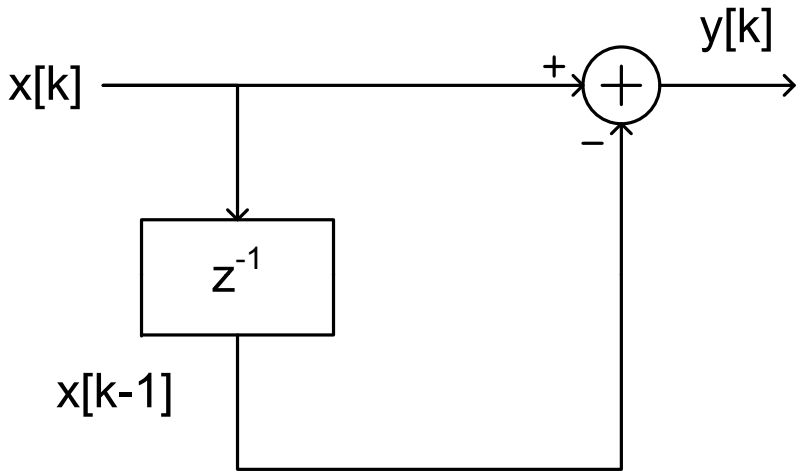
$$y[k \cdot T_S] = \frac{x[k \cdot T_S] - x[(k-1) \cdot T_S]}{T_S}$$

$$y[k] = x[k] - x[k-1]$$



Discrete Time Zero

$$y[k] = x[k] - x[k-1]$$



$$Y(z) = X(z) - z^{-1} \cdot X(z)$$
$$= \left(1 - z^{-1}\right) \cdot X(z)$$

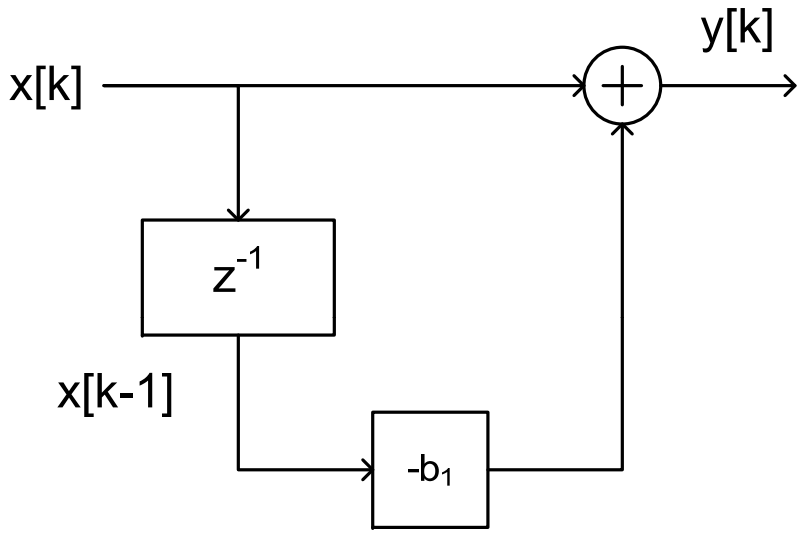
$$H(z) = \frac{Y(z)}{X(z)} = 1 - z^{-1} = \frac{z-1}{z}$$

$$H(1) = 0$$

A zero at dc

Discrete Time Zero

$$y[k] = x[k] - b_1 \cdot x[k-1]$$



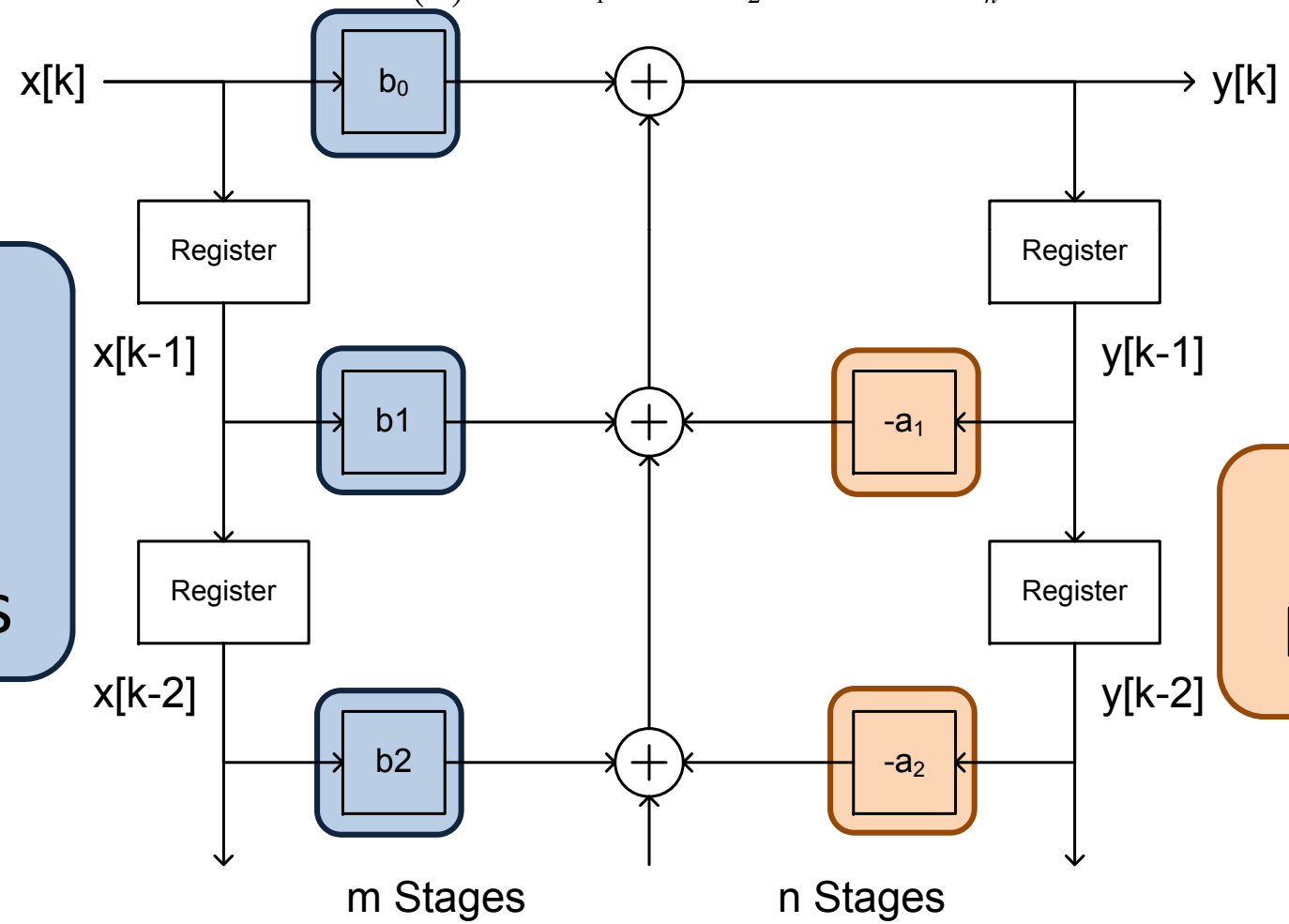
$$Y(z) = X(z) - b_1 \cdot z^{-1} \cdot X(z)$$
$$= \left(1 - b_1 \cdot z^{-1}\right) \cdot X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = 1 - b_1 \cdot z^{-1} = \frac{z - b_1}{z}$$

$$H(b_1) = 1 - b_1 \cdot z^{-1}\Big|_{z=b_1}$$
$$= 1 - \frac{b_1}{b_1}$$
$$= 0$$

Generic Discrete Time System

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 \cdot z^{-1} + \ldots + b_m \cdot z^{-m}}{1 + a_1 \cdot z^{-1} + a_2 \cdot z^{-2} + \ldots + a_n \cdot z^{-n}}$$



Sets Gain And Zeros

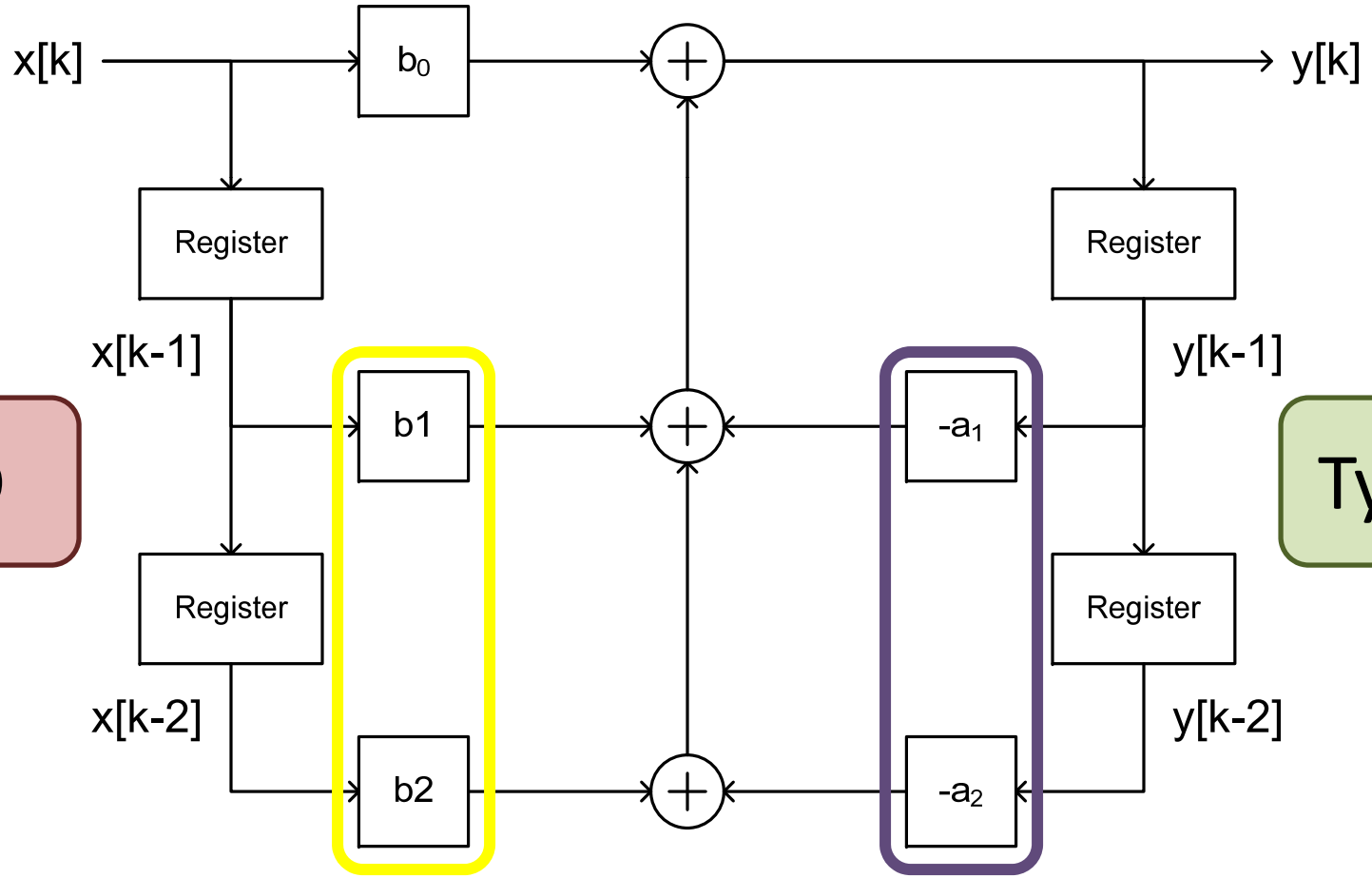
Sets Poles

The "BiQuad"

Two Zeros

$$H(z) = \frac{b_0 + b_1 \cdot z^{-1} + b_2 \cdot z^{-2}}{1 + a_1 \cdot z^{-1} + a_2 \cdot z^{-2}}$$

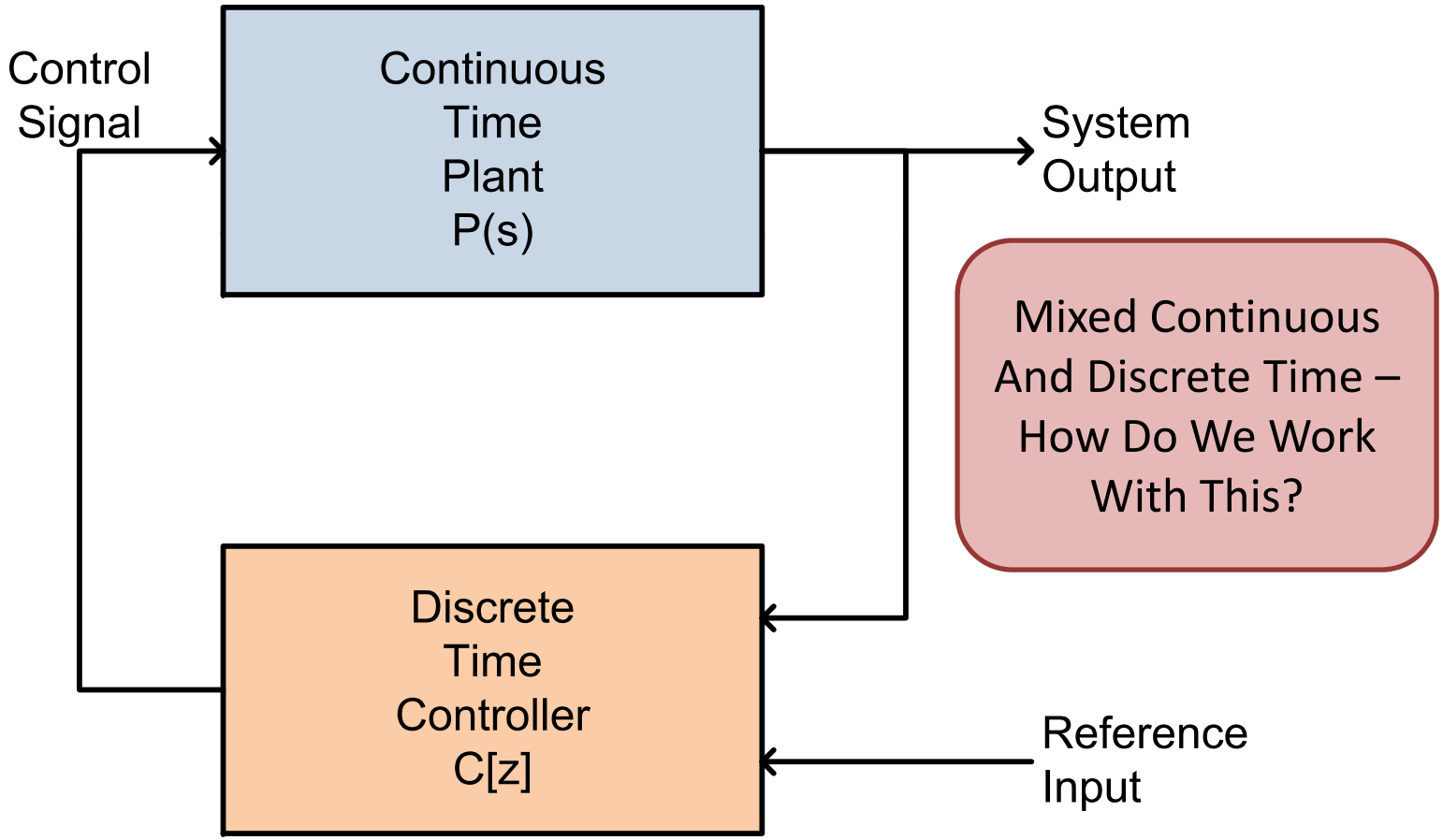
Two Poles



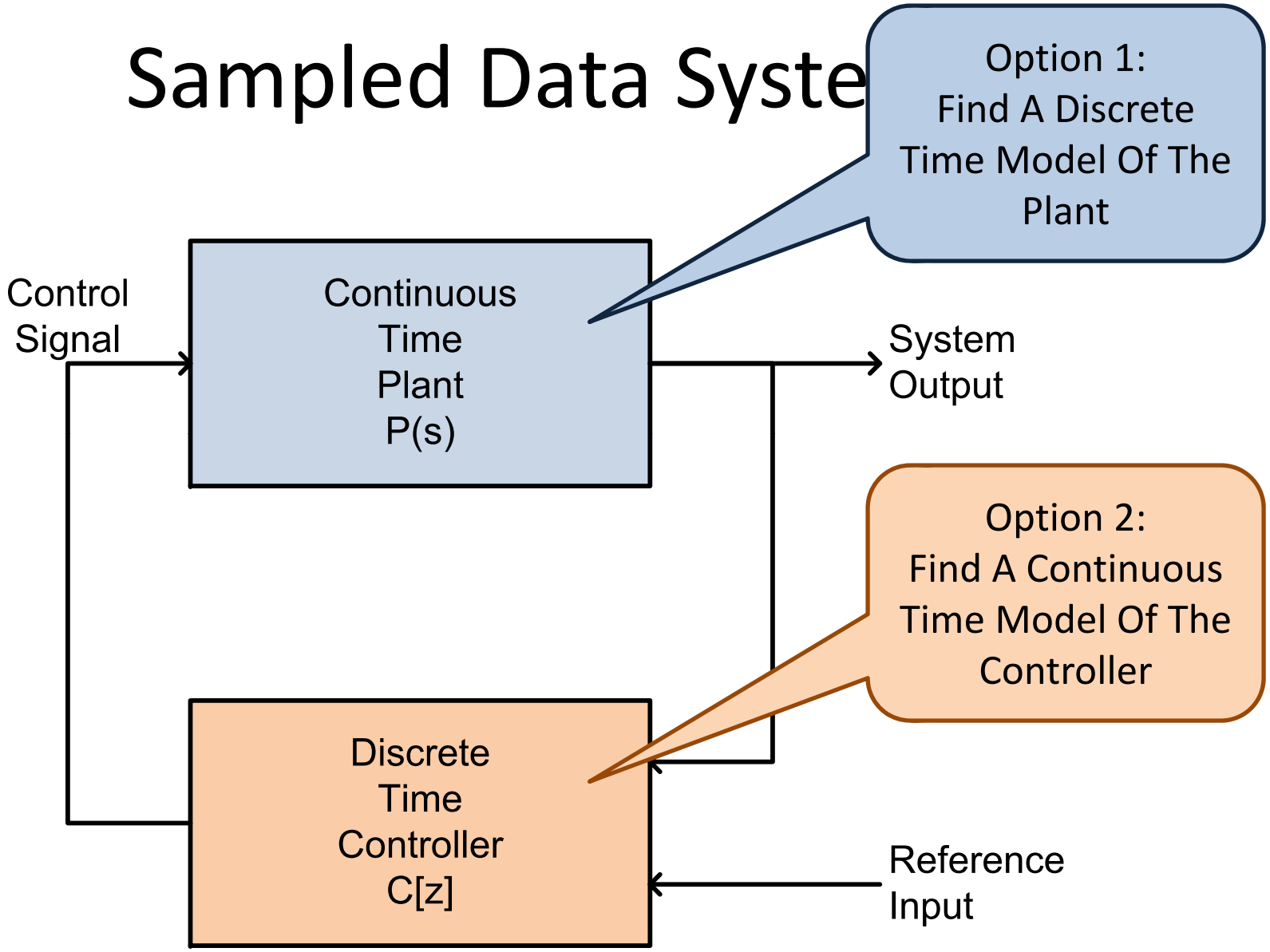
PID

Type 3

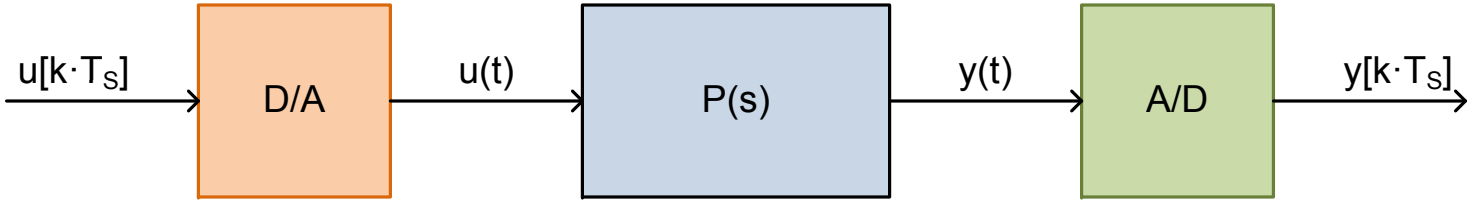
Sampled Data System



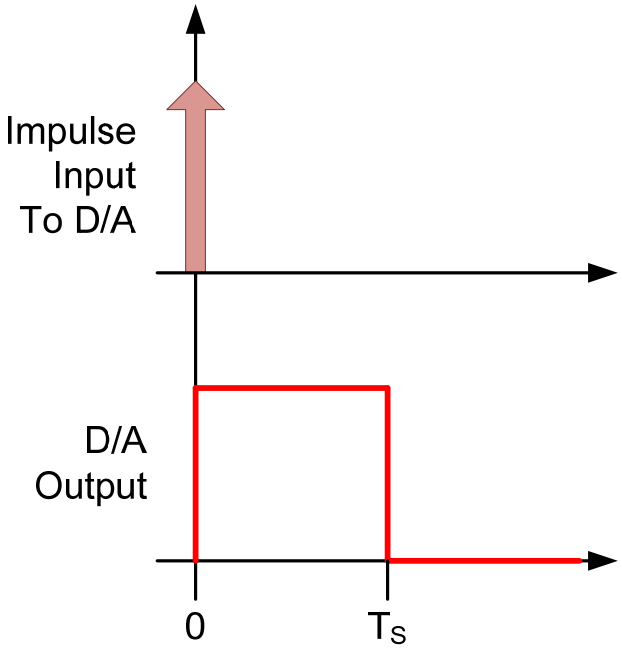
Sampled Data Syste



Discrete Time Model Of A Continuous Time Plant



- Start with the D/A transfer function:



$$D/A \Leftrightarrow ZOH$$

$$h_{D/A}(t) = h_{ZOH}(t) = 1(t) - 1(t - T_S)$$

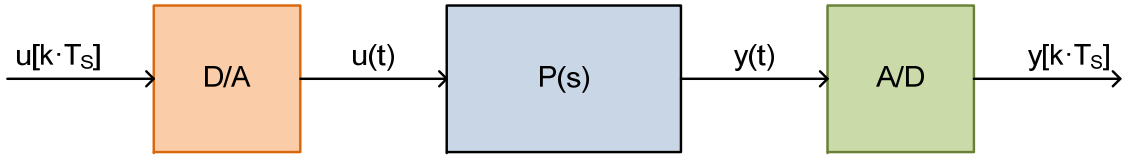
$$\mathcal{L}\{h_{ZOH}(t)\} = \mathcal{L}\{1(t)\} - \mathcal{L}\{1(t - T_S)\}$$

$$= \frac{1}{s} - \frac{1}{s} \cdot e^{-s \cdot T_S}$$

$$H_{ZOH}(s) = \frac{1 - e^{-s \cdot T_s}}{s}$$

Discrete Time Model Of A Continuous Time Plant

$$Y(s) = P(s) \cdot H_{ZOH}(s)$$
$$= \left(\frac{1 - e^{-s \cdot T_s}}{s} \right) \cdot P(s)$$



$$y(k \cdot T_s) = \mathcal{L}^{-1} \left\{ \left(1 - e^{-s \cdot T_s} \right) \cdot \frac{P(s)}{s} \right\} \Bigg|_{t = k \cdot T_S}$$

$$Y(z) = \mathcal{Z} \{ y(k \cdot T_s) \}$$
$$= \mathcal{Z} \left\{ \mathcal{L}^{-1} \left\{ \frac{P(s)}{s} \right\} \right\} - \mathcal{Z} \left\{ \mathcal{L}^{-1} \left\{ e^{-s \cdot T_s} \cdot \frac{P(s)}{s} \right\} \right\}$$

$$Y(z) = \left(1 - z^{-1} \right) \cdot \mathcal{Z} \left\{ \mathcal{L}^{-1} \left\{ \frac{P(s)}{s} \right\} \Bigg|_{t = k \cdot T_S} \right\}$$

Example

$$P(s) = \frac{a}{s+a}$$

$$\frac{P(s)}{s} = \frac{a}{s\cdot(s+a)} = \frac{1}{s} - \frac{1}{s+a}$$

$$\mathcal{L}^{-1}\left\{\frac{P(s)}{s}\right\} = 1(t) - e^{-a\cdot t}\cdot 1(t)$$

$$\mathcal{Z}\left\{\mathcal{L}^{-1}\left\{\frac{P(s)}{s}\right\}\Big|_{t=k\cdot T_S}\right\} = \mathcal{Z}\left\{1(k\cdot T_S) - e^{-a\cdot k\cdot T_S}\cdot 1(k\cdot T_S)\right\}$$

$$= \frac{z}{z-1} - \frac{z}{z-e^{-a\cdot T_S}}$$

$$= \frac{z\cdot\left(1-e^{-a\cdot T_S}\right)}{(z-1)\cdot\left(z-e^{-a\cdot T_S}\right)}$$

$$Y(z) = \left(1-z^{-1}\right)\cdot\frac{z\cdot\left(1-e^{-a\cdot T_S}\right)}{(z-1)\cdot\left(z-e^{-a\cdot T_S}\right)}$$

$$= \frac{z-1}{z}\cdot\frac{z\cdot\left(1-e^{-a\cdot T_S}\right)}{(z-1)\cdot\left(z-e^{-a\cdot T_S}\right)}$$

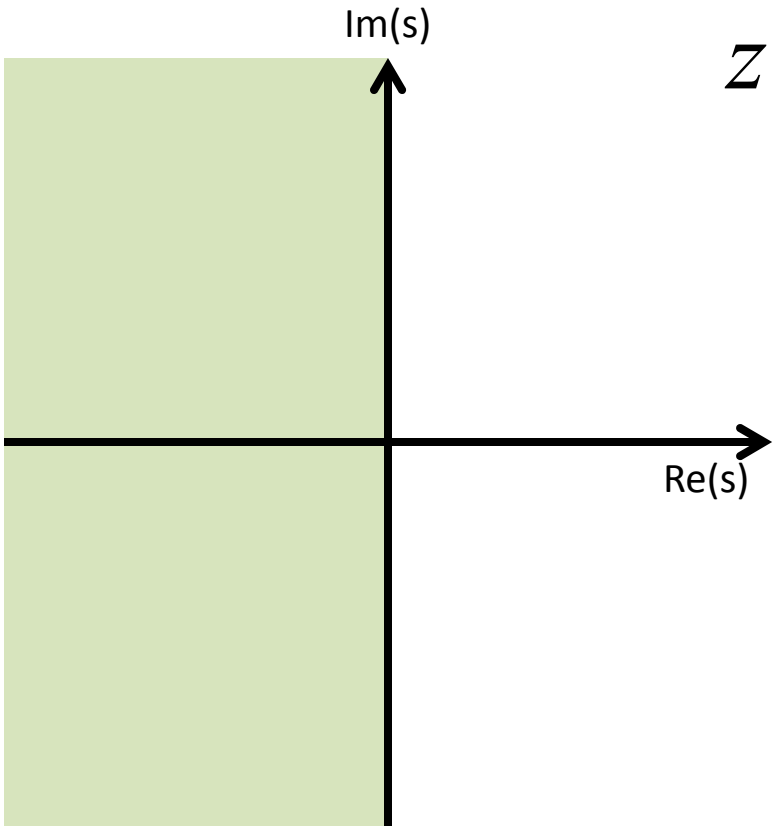
$$= \frac{1-e^{-a\cdot T_S}}{z-e^{-a\cdot T_S}}$$

Single Pole Response!

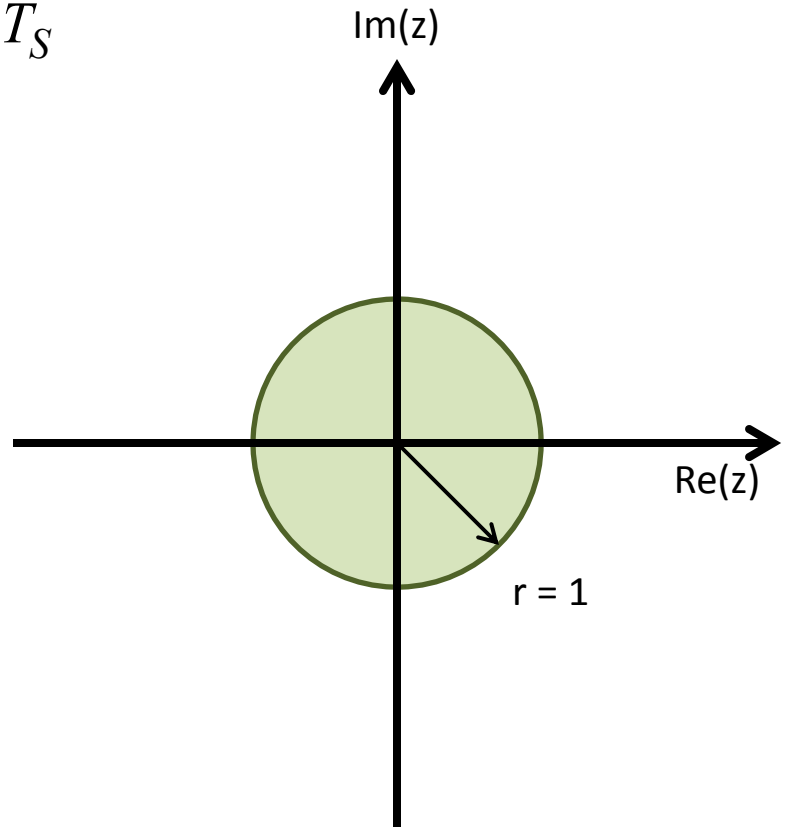
Example taken from *Digital Control of Dynamic Systems*, 2nd ed., Franklin, Powell, and Workman, Ellis-Kagle Press

s Plane ⇔ *z* Plane

$$z = e^{s \cdot T_S}$$

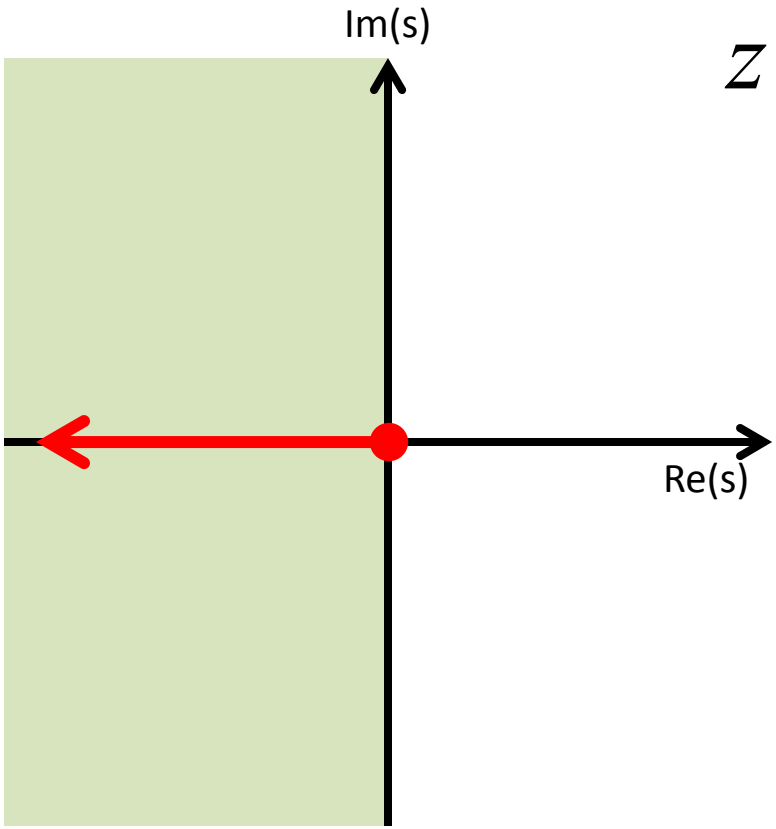


Stable: Poles In LHP

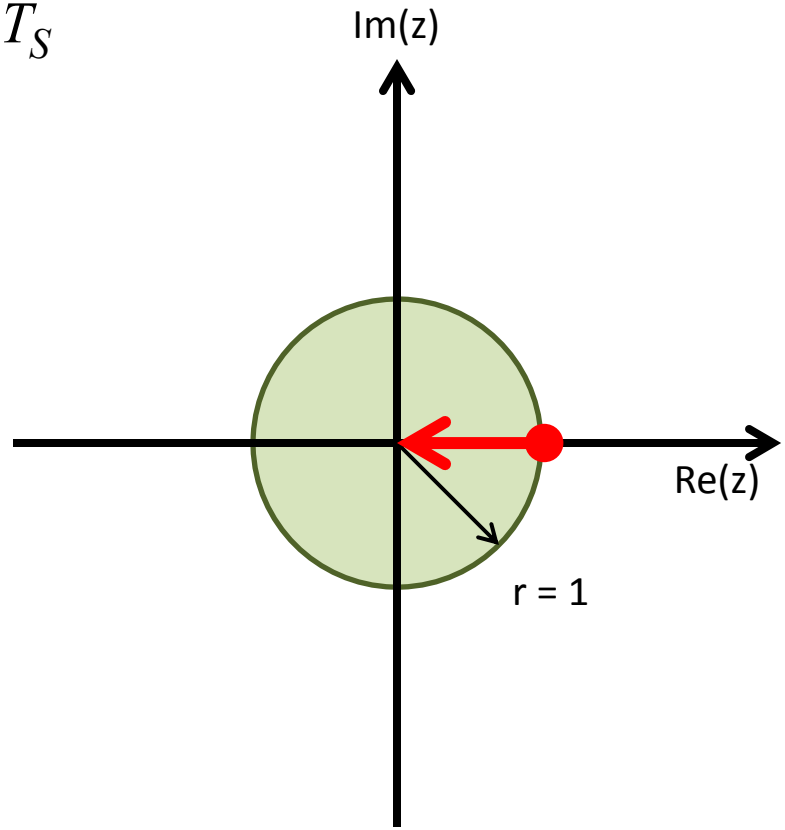


Stable: Poles Inside Unit Circle

s Plane ⇔ *z* Plane



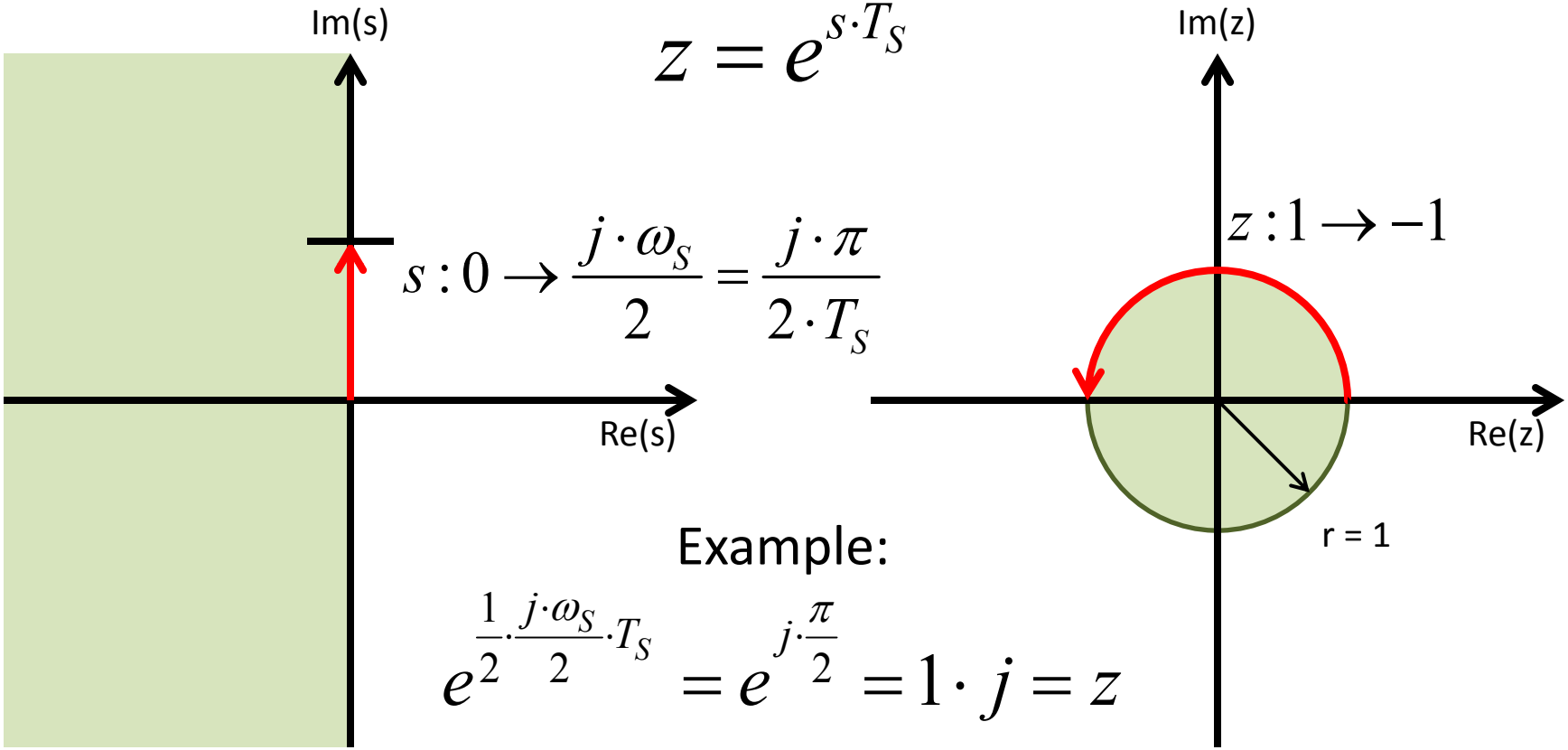
$$z = e^{s \cdot T_S}$$



DC: *s* = 0

DC: *z* = 1

s Plane ⇔ *z* Plane



$$z = e^{s \cdot T_S}$$

Example:

$$e^{\frac{1}{2} \cdot \frac{j \cdot \omega_S}{2} \cdot T_S} = e^{j \cdot \frac{\pi}{2}} = 1 \cdot j = z$$

$$\frac{F_S}{2} = \frac{1}{2 \cdot T_S} = \frac{\omega_S}{4 \cdot \pi} \Rightarrow \text{"Infinite Frequency"}$$

Discrete Equivalents

- Consider the transfer function:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s}$$

- This becomes the differential equation:

$$s \cdot Y(s) = X(s)$$

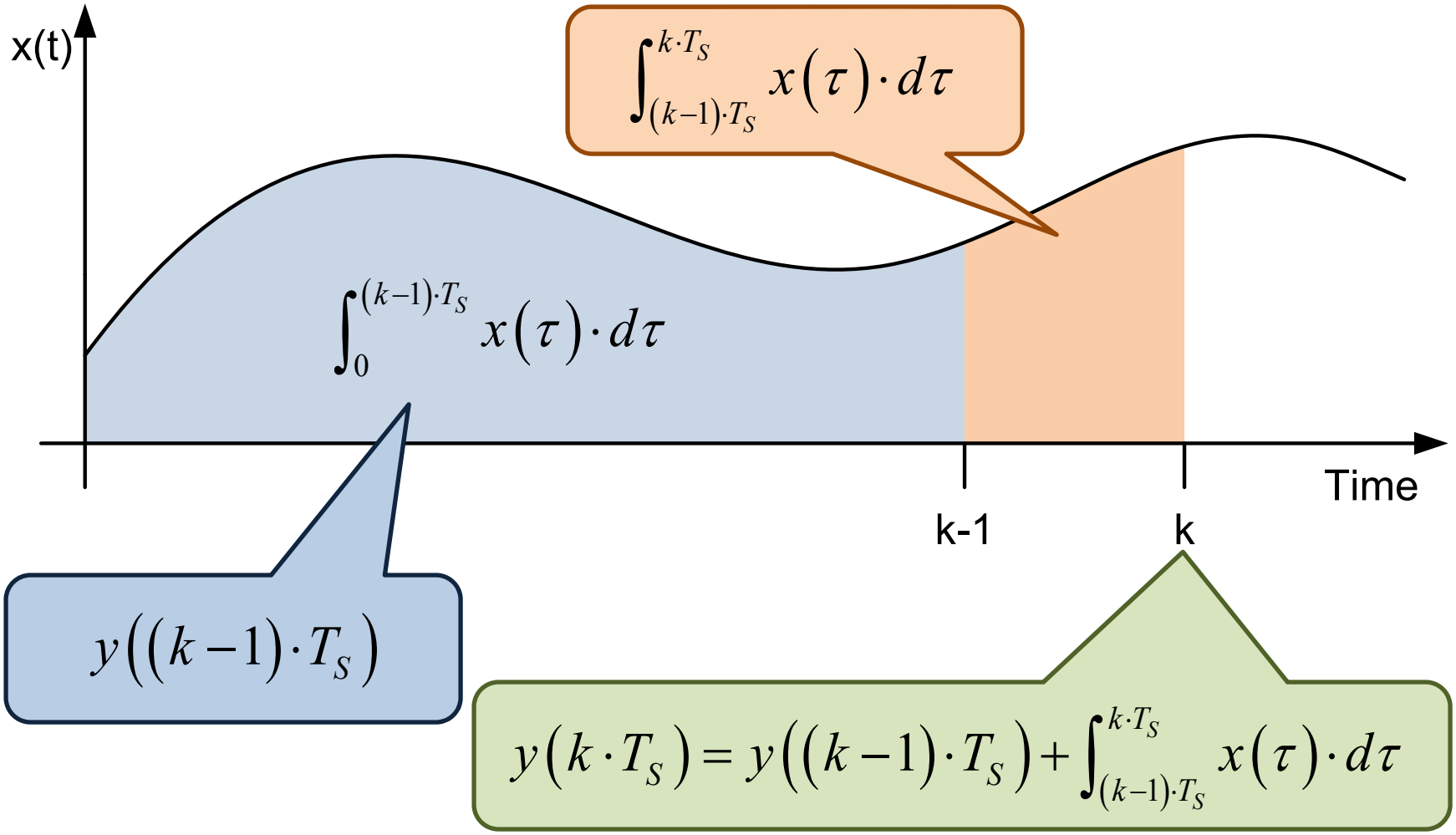
$$\frac{dy(t)}{dt} = x(t)$$

- Solve by integrating:

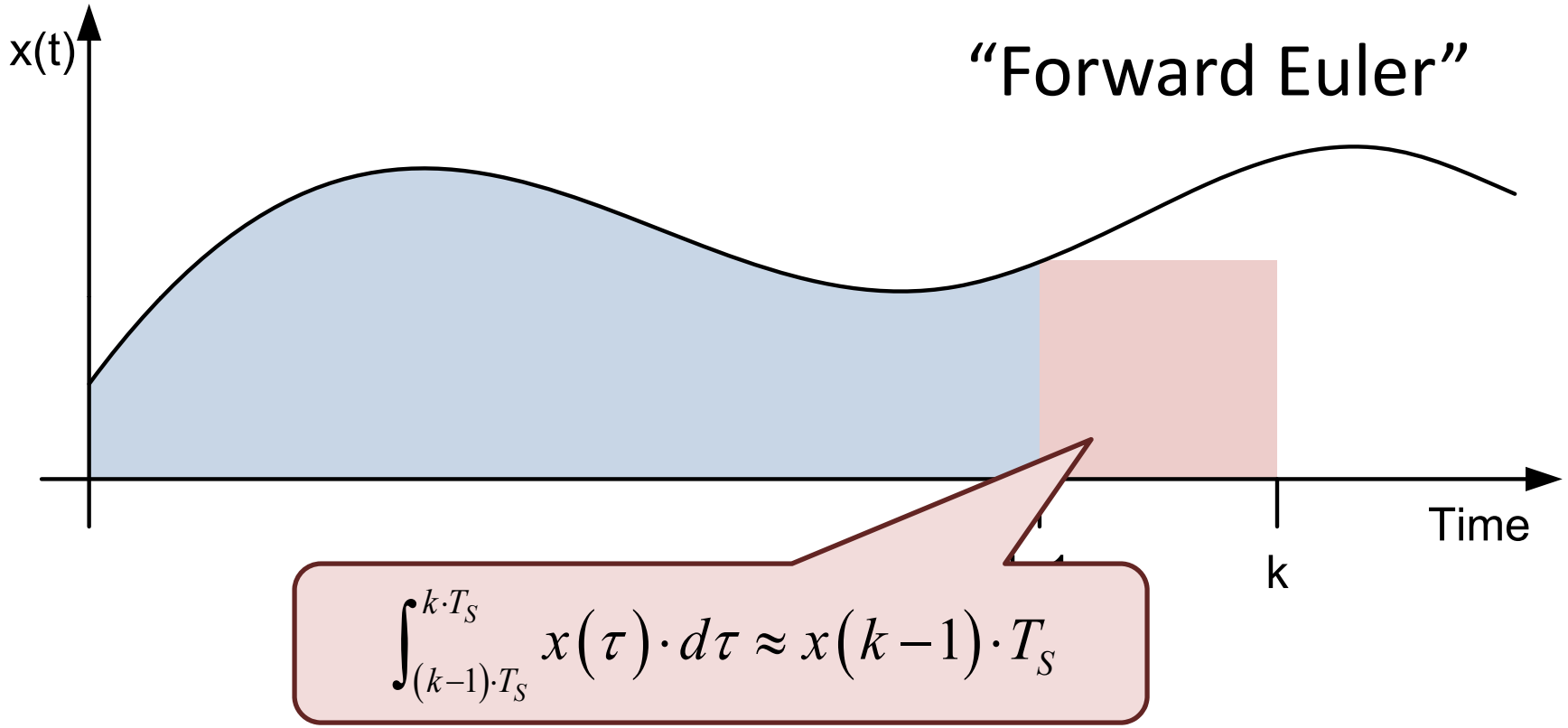
$$y(t) = \int_0^t x(\tau) \cdot d\tau$$

- How to approximate in discrete time?

Discrete Equivalents



Discrete Equivalents



$$y(k\cdot T_S) \approx y(k-1) + x(k-1)\cdot T_S$$

Discrete Equivalents

$$y(k \cdot T_S) \approx y(k-1) + x(k-1) \cdot T_S$$

$$Y(z) = z^{-1} \cdot Y(z) + z^{-1} \cdot T_S \cdot X(z)$$

$$Y(z) - z^{-1} Y(z) = z^{-1} \cdot T_S \cdot X(z)$$

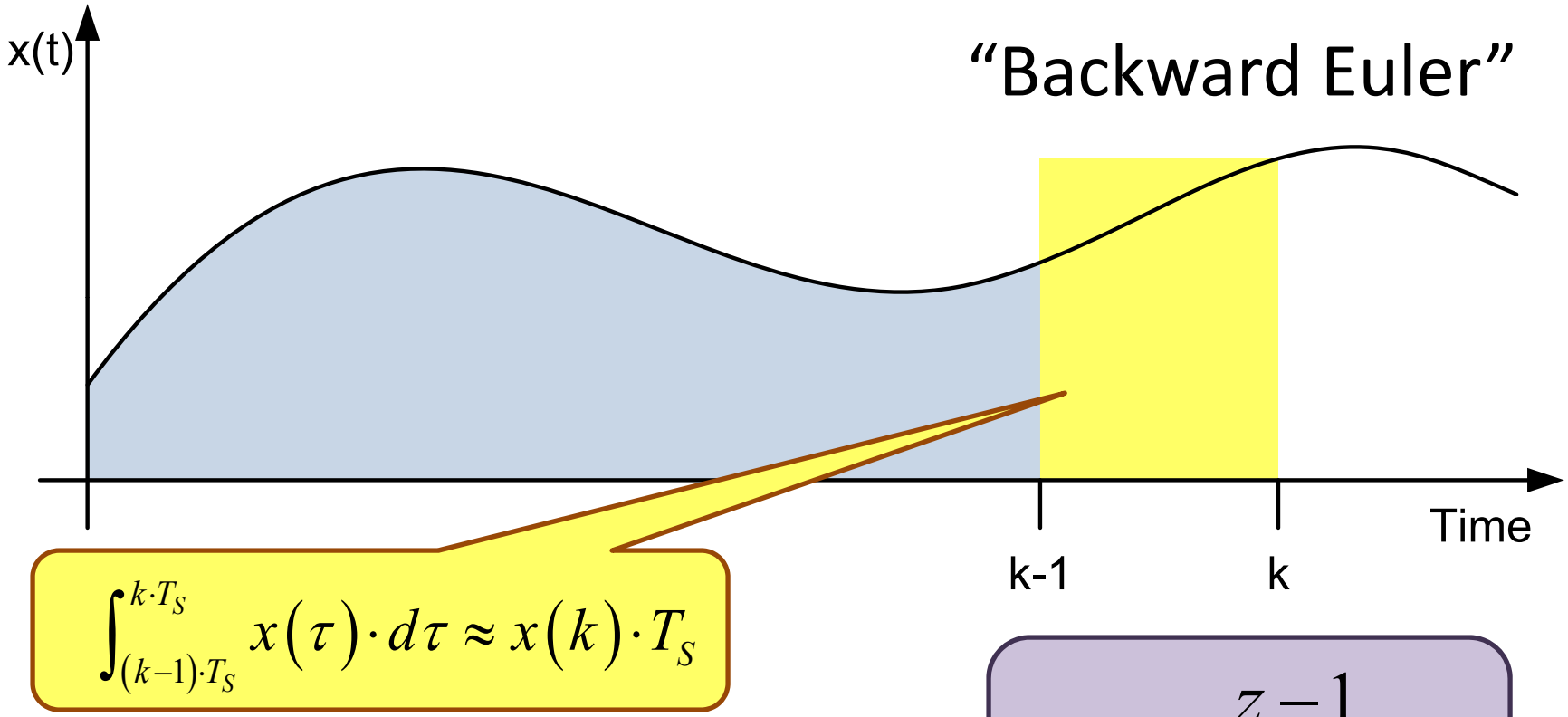
$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1} \cdot T_S}{1 - z^{-1}}$$

$$= \frac{1}{\frac{z}{T_S} - \frac{1}{T_S}} = \frac{1}{\frac{z-1}{T_S}}$$

$$H(z) = H(s)\Big|_{s=\frac{z-1}{T_S}}$$

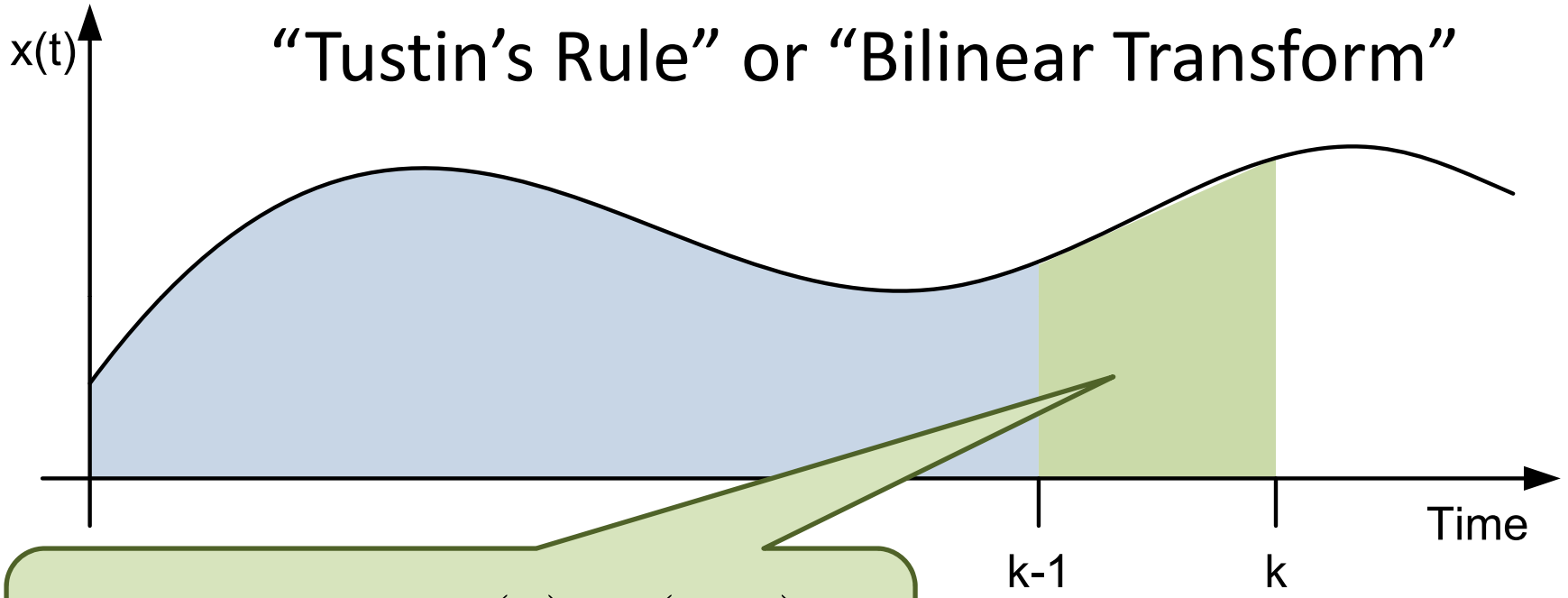
$$s \Leftrightarrow \frac{z-1}{T_S}$$

Discrete Equivalents



Discrete Equivalents

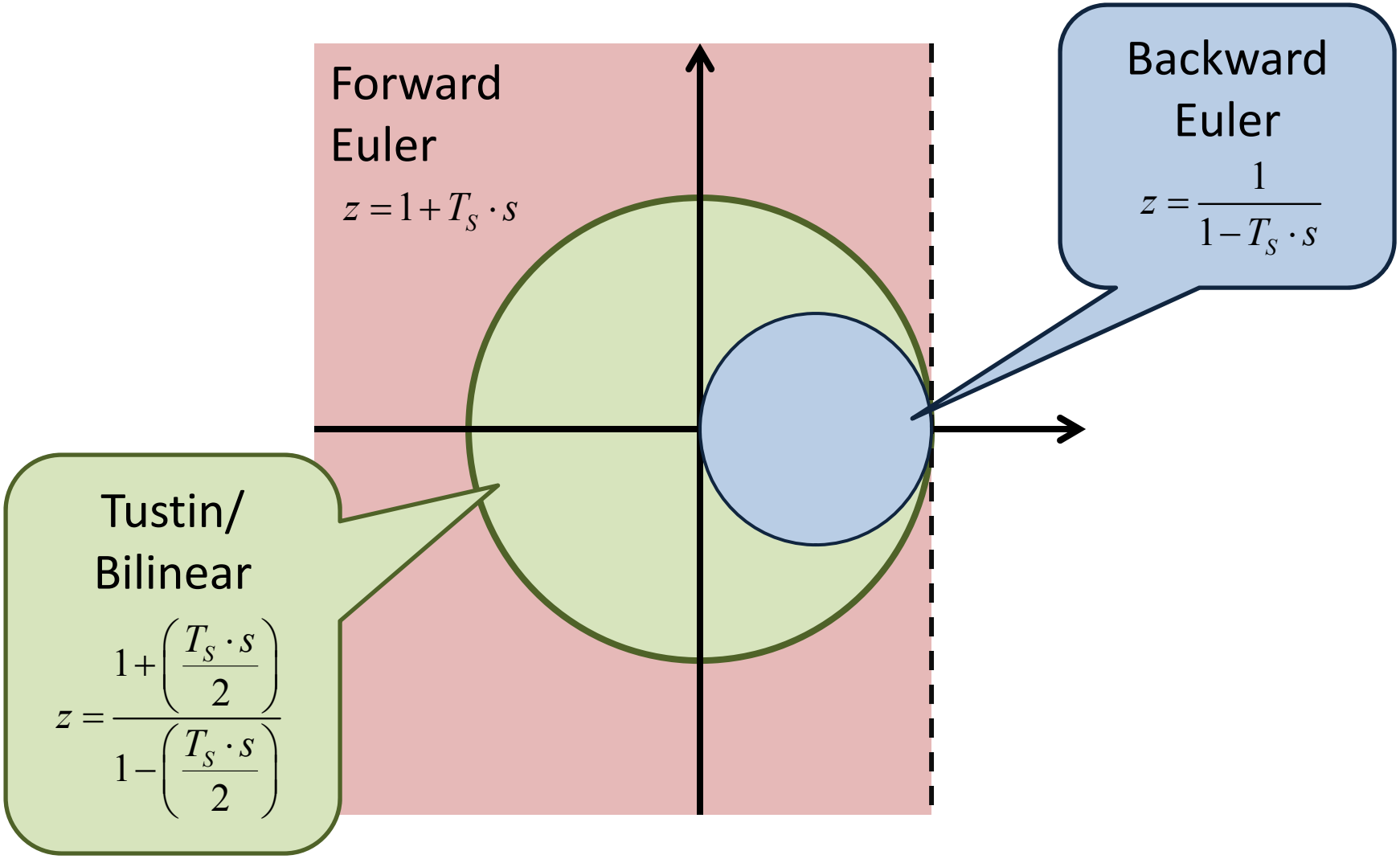
"Tustin's Rule" or "Bilinear Transform"



$$\int_{(k-1)\cdot T_S}^{k\cdot T_S} x(\tau)\cdot d\tau \approx \frac{x(k)+x(k-1)}{2}\cdot T_S$$

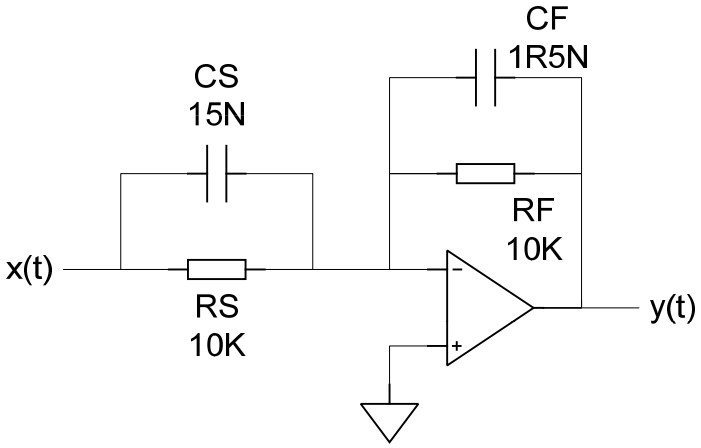
$$s \Leftrightarrow \frac{2}{T_S}\cdot\frac{z-1}{z+1}$$

Mapping And Stability



Example: Continuous To Discrete Time Conversion

- Wanted: Lead-Lag Circuit
  - F_{ZERO} = 1 kHz, F_{POLE} = 10 kHz, K_{DC} = -1



$$H(s) = \frac{Y(s)}{X(s)} = -\frac{Z_F(s)}{Z_S(s)}$$

$$Z_F(s) = R_F \parallel \frac{1}{s \cdot C_F} = \frac{R_F}{1 + s \cdot R_F \cdot C_F}$$

$$Z_F(s) = R_S \parallel \frac{1}{s \cdot C_S} = \frac{R_S}{1 + s \cdot R_S \cdot C_S}$$

$$H(s) = -\frac{\frac{R_F}{1 + s \cdot R_F \cdot C_F}}{\frac{R_S}{1 + s \cdot R_S \cdot C_S}} = -\frac{R_F}{R_S} \cdot \frac{1 + s \cdot R_S \cdot C_S}{1 + s \cdot R_F \cdot C_F}$$

$$K_{DC} = -1 \Rightarrow R_F = R_S = 10\ k\Omega$$

$$C_S = \frac{1}{2 \cdot \pi \cdot F_{ZERO} \cdot R_S} = 15.9\ \text{nF} \Rightarrow 15\ \text{nF}$$

$$F_{ZERO} = \frac{1}{2 \cdot \pi \cdot R_S \cdot C_S} = 1.06\ \text{kHz}$$

$$C_F = \frac{1}{2 \cdot \pi \cdot F_{POLE} \cdot R_F} = 1.59\ \text{nF} \Rightarrow 1.5\ \text{nF}$$

$$F_{POLE} = \frac{1}{2 \cdot \pi \cdot R_F \cdot C_F} = 10.6\ \text{kHz}$$

Example: Continuous To Discrete Time Conversion

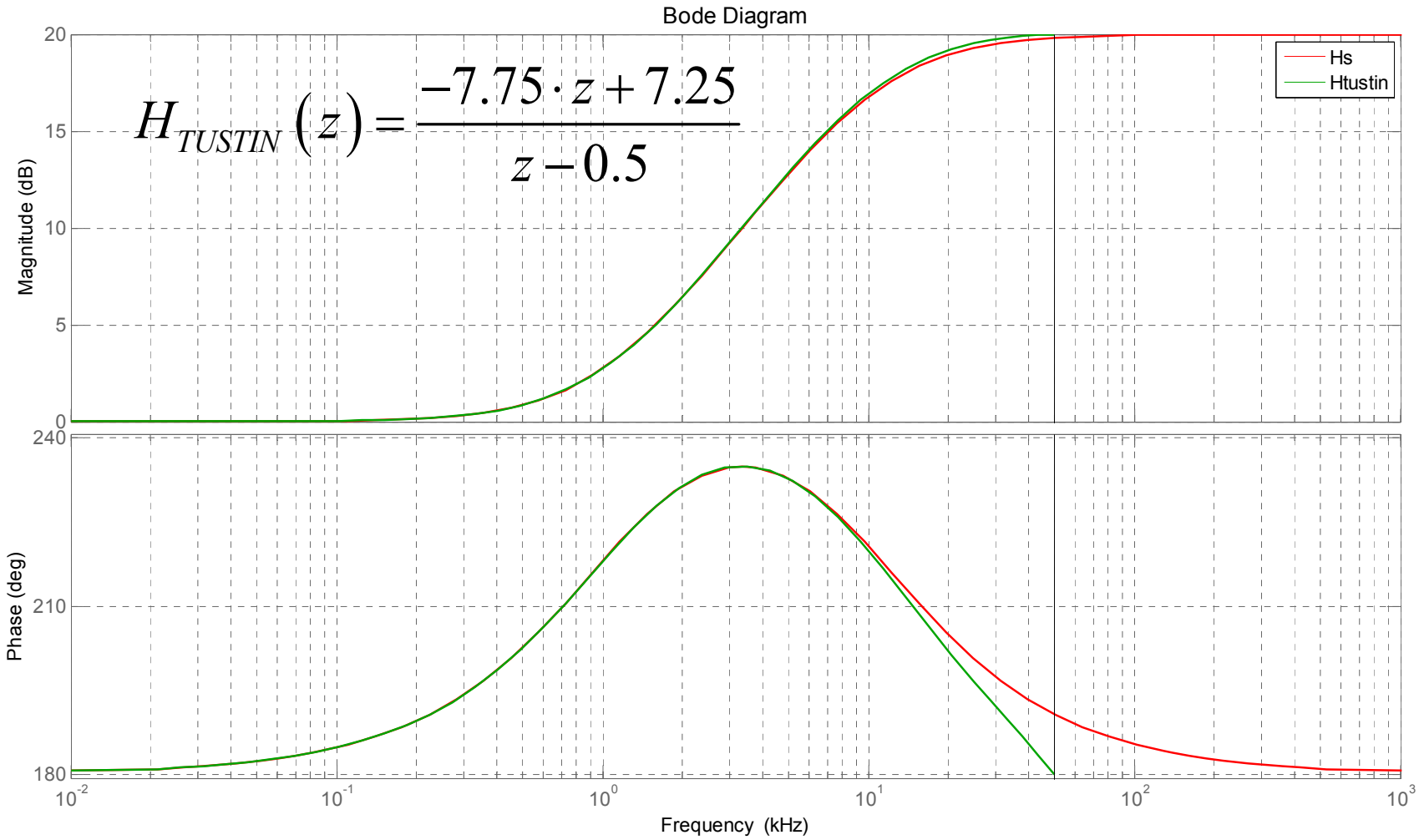
$$F_{SAMPLE} = 100\ \text{kHz} \qquad T_{SAMPLE} = \frac{1}{F_{SAMPLE}} = 10\ \mu\text{s}$$

$$s \Leftrightarrow \frac{2}{T_S} \cdot \frac{z-1}{z+1}$$

$$H(s) = K_{DC} \cdot \frac{1 + \frac{s}{\omega_{ZERO}}}{1 + \frac{s}{\omega_{POLE}}}$$

$$H_{TUSTIN}(z) = K_{DC} \cdot \frac{1 + \frac{\frac{2}{T_S} \cdot \frac{z-1}{z+1}}{\omega_{ZERO}}}{1 + \frac{\frac{2}{T_S} \cdot \frac{z-1}{z+1}}{\omega_{POLE}}}$$

Example: Continuous To Discrete Time Conversion



EMBEDDED POWER LABS

Example: Continuous To Discrete Time Conversion

- What about direct pole-zero mapping?

$$H(z) = K_Z \cdot \frac{z - z_{ZERO}}{z - z_{POLE}}$$

$$z_{ZERO} = e^{-\omega_{ZERO} \cdot T_{SAMPLE}} = 0.9355$$

$$z_{POLE} = e^{-\omega_{POLE} \cdot T_{SAMPLE}} = 0.5134$$

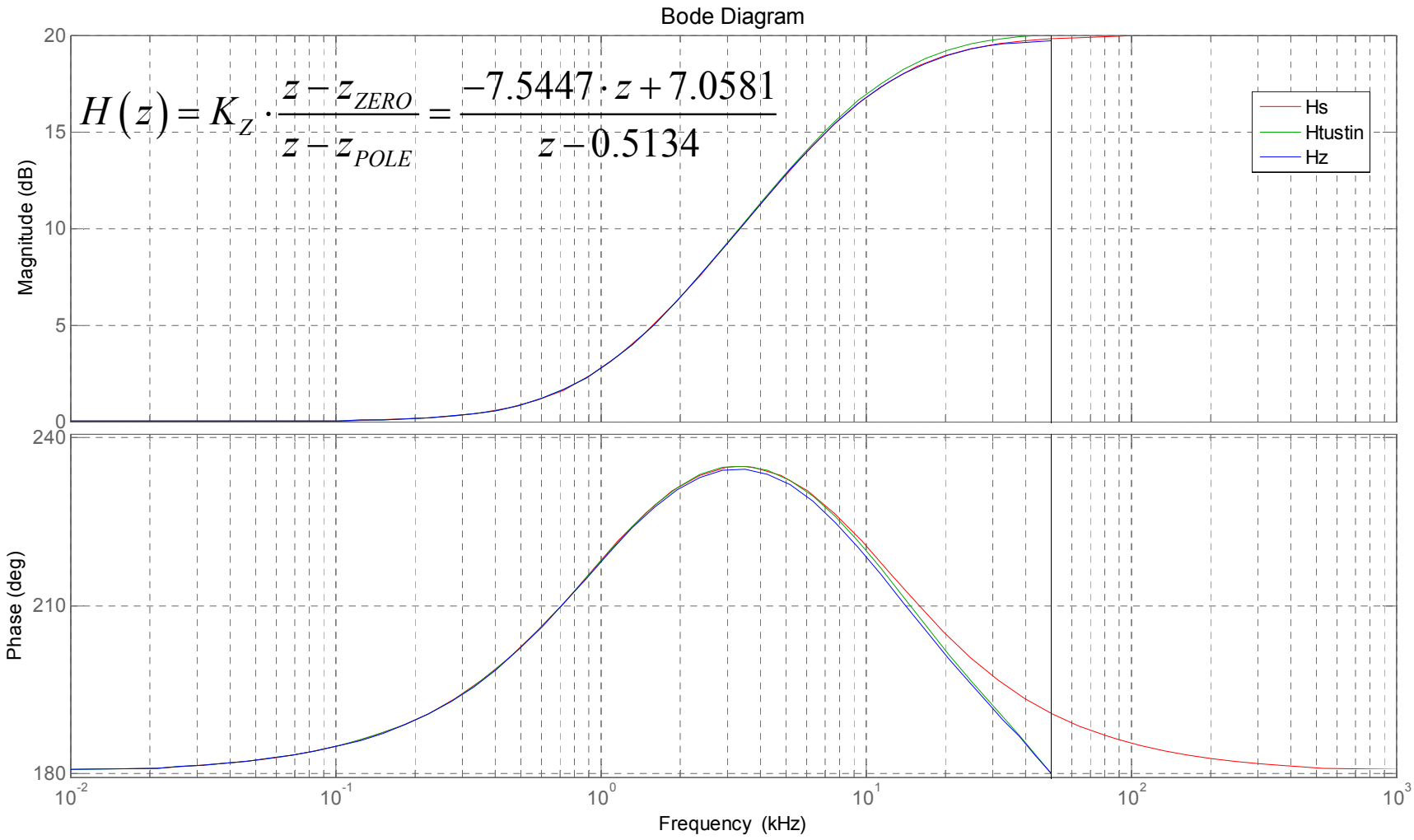
$$H(1) = -1 = K_Z \cdot \frac{1 - z_{ZERO}}{1 - z_{POLE}}$$

$$K_Z = -\frac{1 - z_{POLE}}{1 - z_{ZERO}} = -7.5447$$

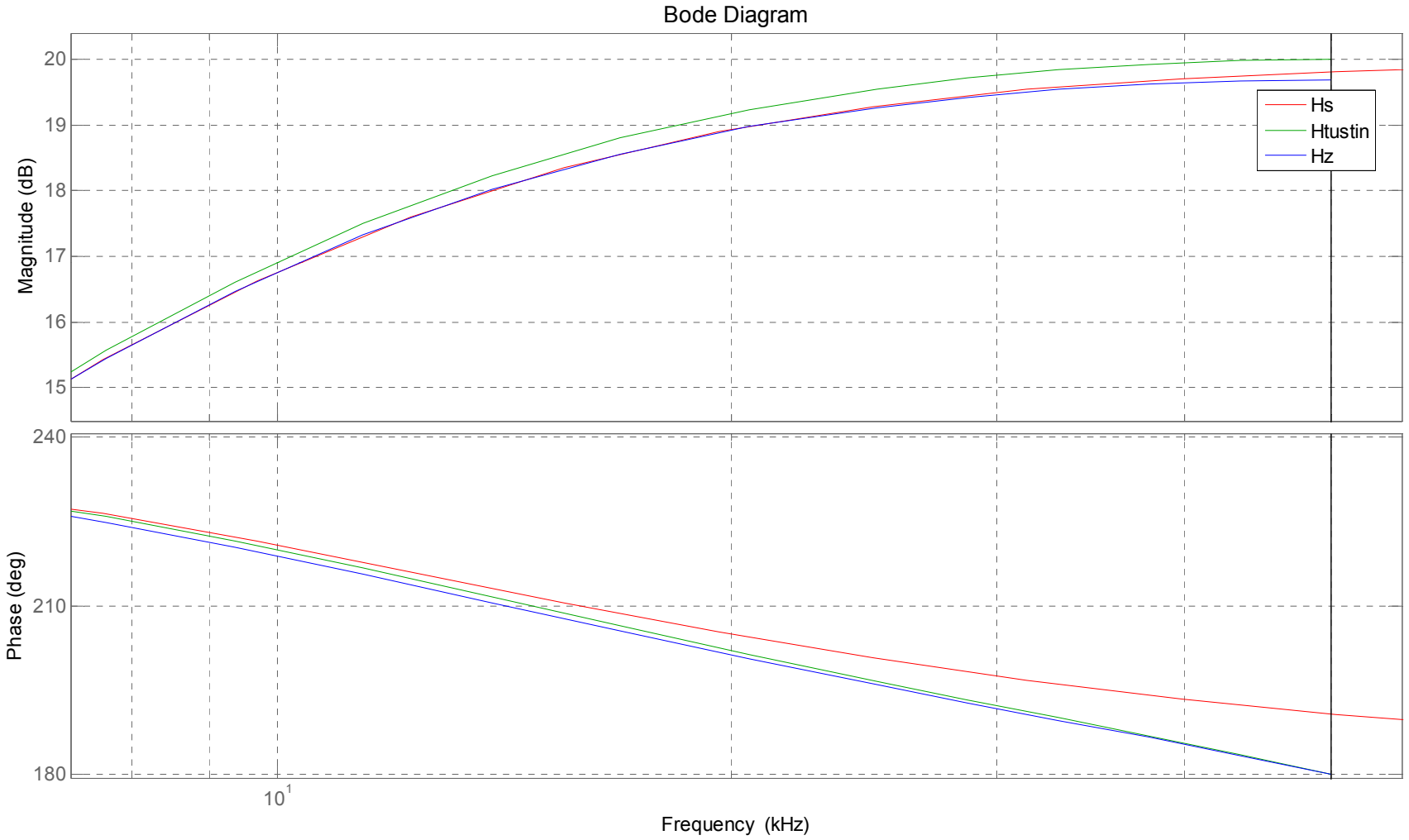
$$H(z) = -7.5447 \cdot \frac{z - 0.9355}{z - 0.5134}$$

$$= \frac{-7.5447 \cdot z + 7.0581}{z - 0.5134}$$

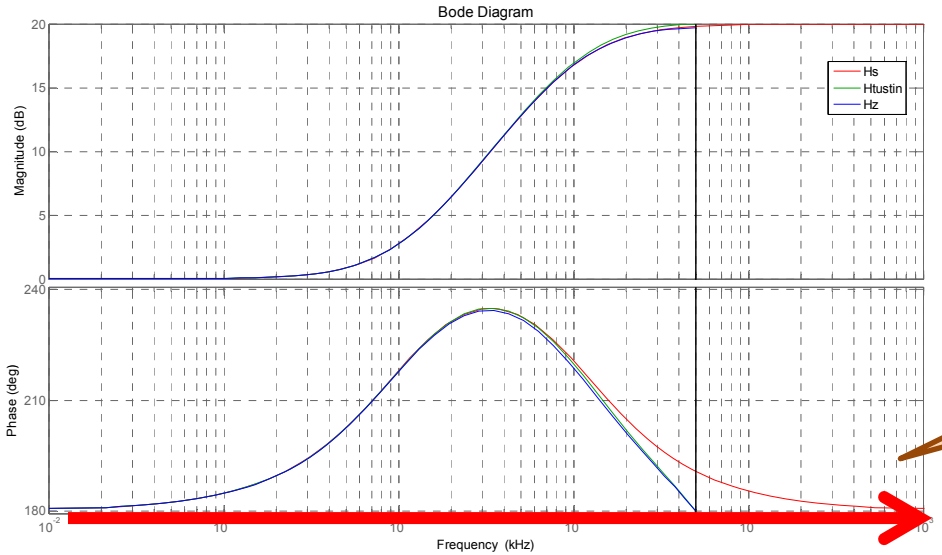
Example: Continuous To Discrete Time Conversion



Example: Continuous To Discrete Time Conversion

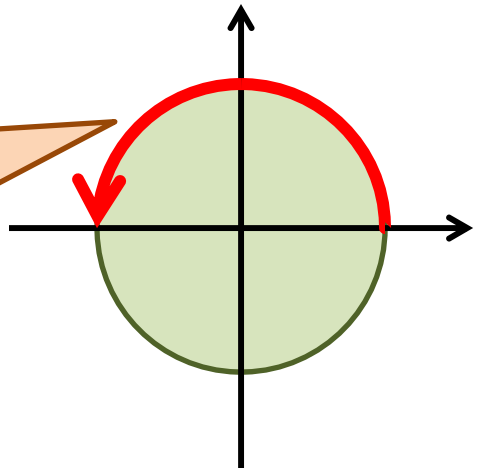


About The Bode Plots



As frequency on the Bode plot moves from low to high frequency…

…evaluation moves along unit circle from +1 to -1



Bode plot magnitude is found by evaluating:

$$\left| H\left(z = e^{j \cdot \omega \cdot T_{SAMPLE}} \right) \right|$$

EMBEDDED POWER LABS

Example: Continuous To Discrete Time Conversion

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-7.5447 \cdot z + 7.0581}{z - 0.5134} \cdot \frac{z^{-1}}{z^{-1}}$$

$$\frac{Y(z)}{X(z)} = \frac{-7.5447 + 7.0581 \cdot z^{-1}}{1 - 0.5134 \cdot z^{-1}}$$

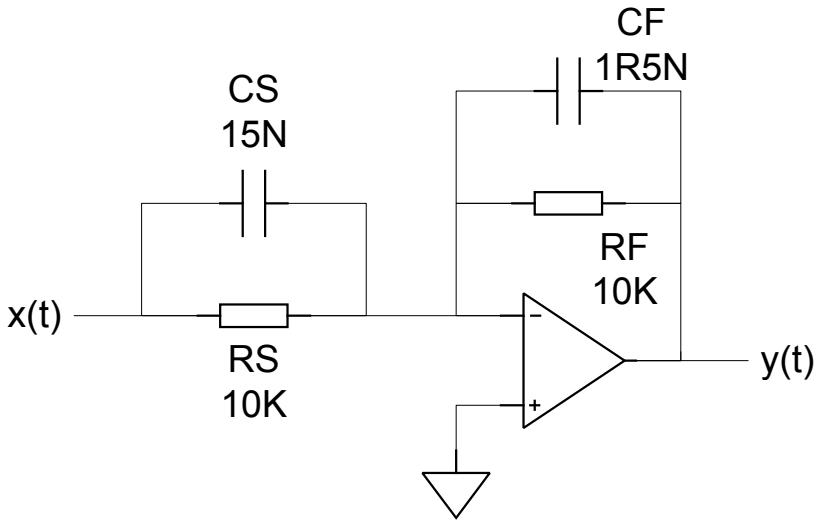
$$Y(z) - 0.5134 \cdot z^{-1} \cdot Y(z) = -7.5447 \cdot X(z) + 7.0581 \cdot z^{-1} \cdot X(z)$$

$$y[k] - 0.5134 \cdot y[k-1] = -7.5447 \cdot x[k] + 7.0581 \cdot x[k-1]$$

$$y[k] = 0.5134 \cdot y[k-1] - 7.5447 \cdot x[k] + 7.0581 \cdot x[k-1]$$

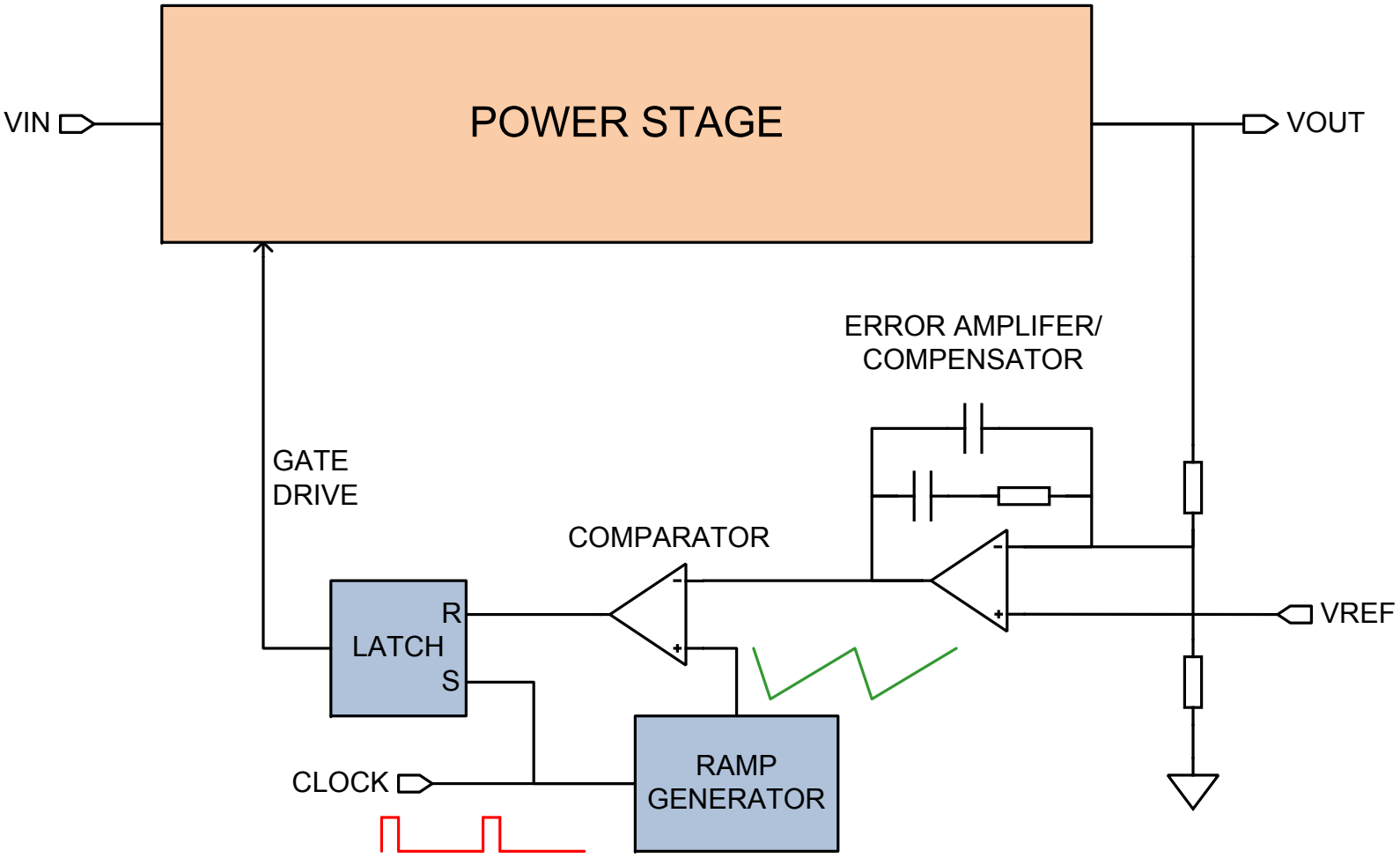
Example: Continuous To Discrete Time Conversion

```
LeadLag(Xnew){
   Ynew = 0.5134*Yold-7.7447*Xnew _
   +7.0581*Xold;

   Xold = Xnew;
   Yold = Ynew;

   return(Ynew);
   }
```

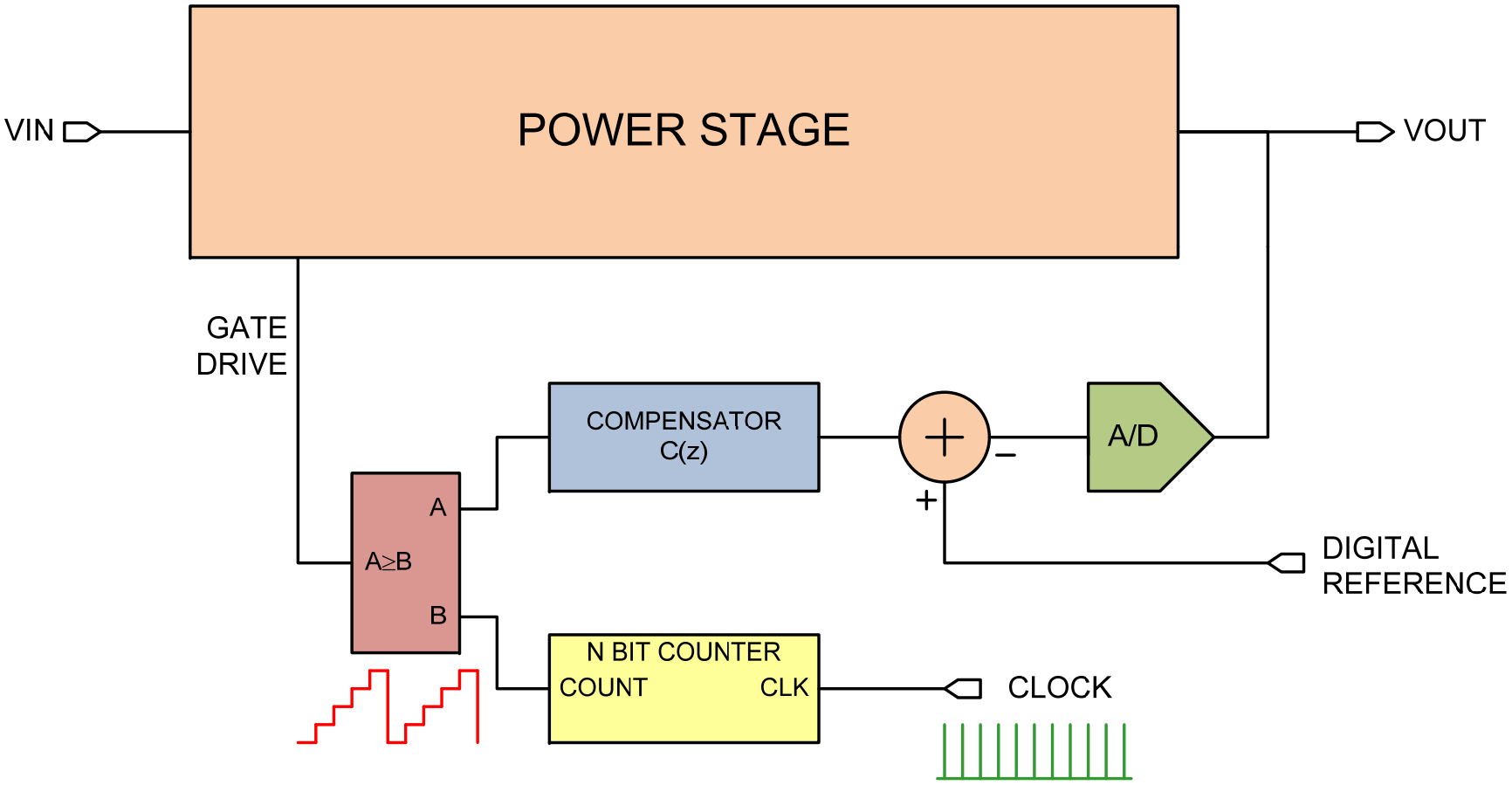


Implementing Digital Control For A Power Supply

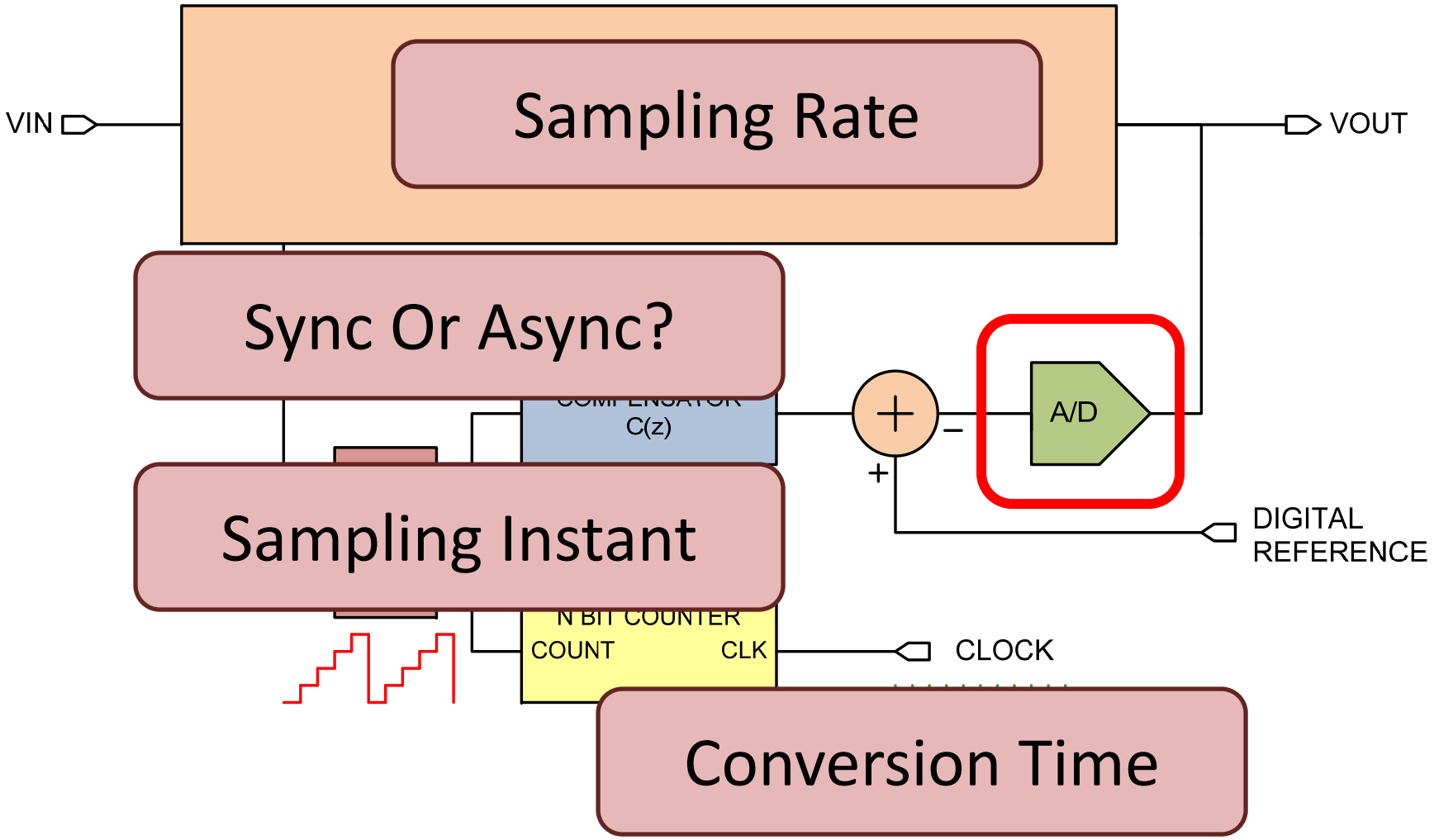
Power Supply With Analog Controller



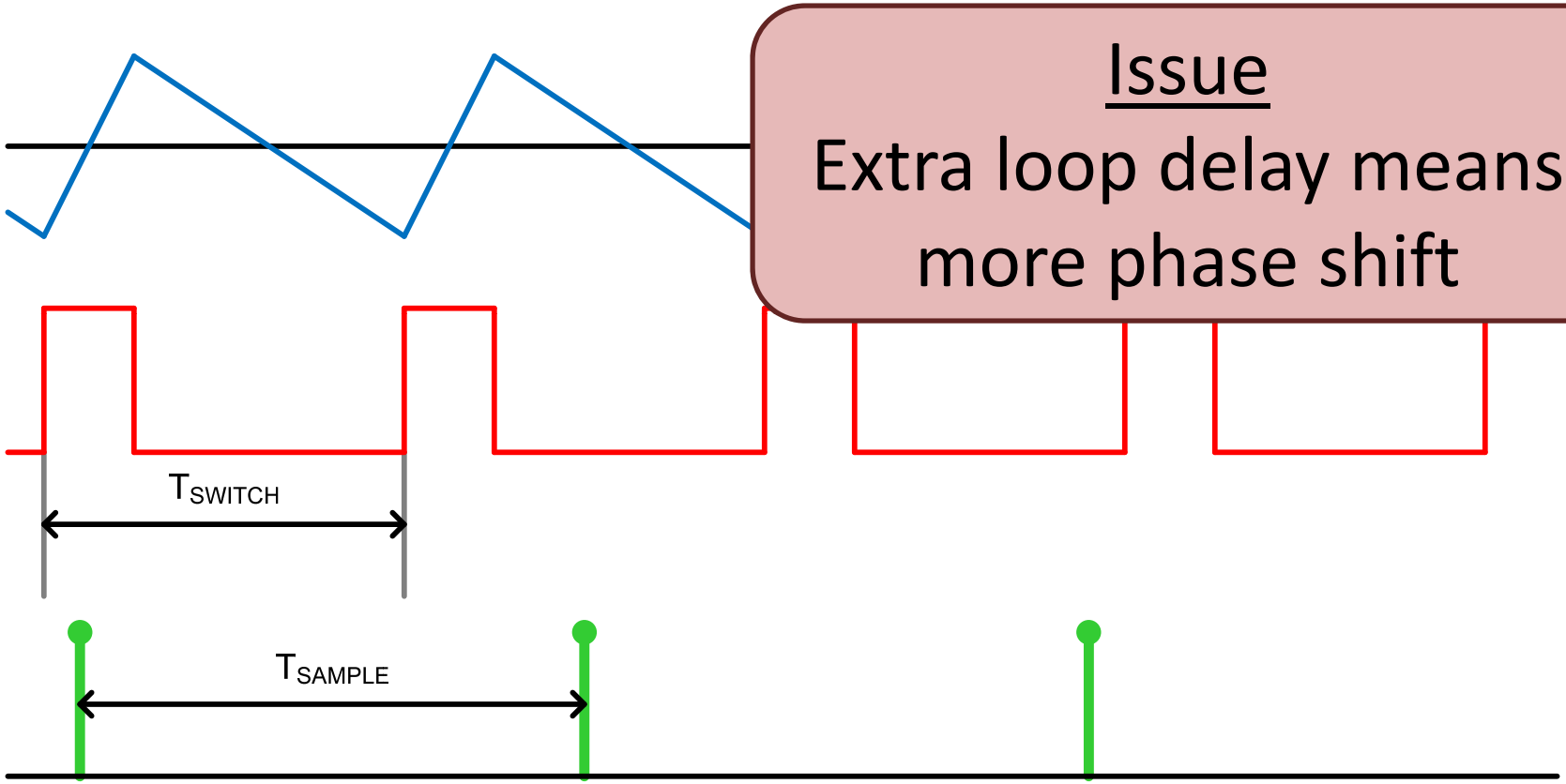
Straightforward Digital Conversion



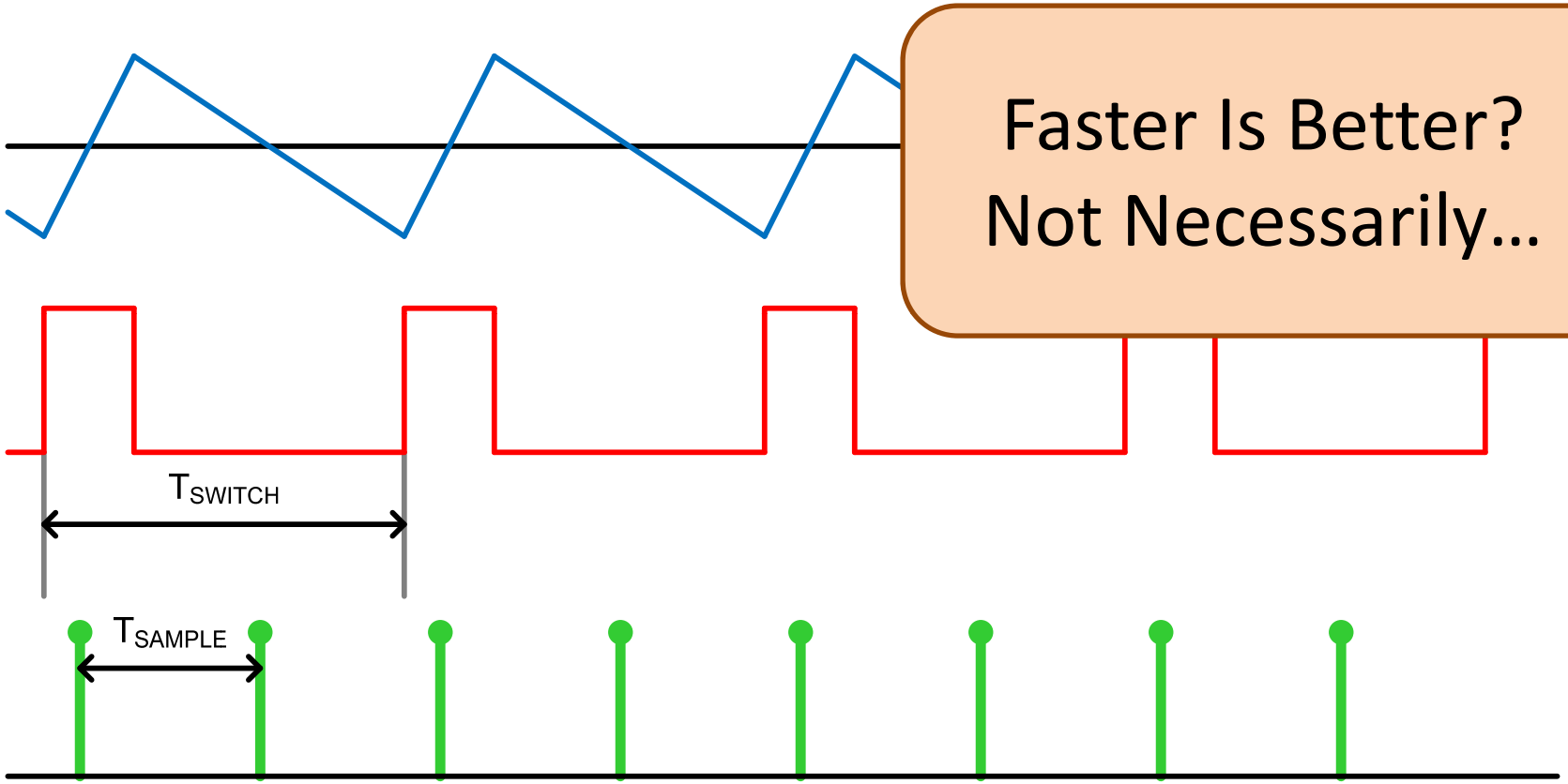
Sampling Issues



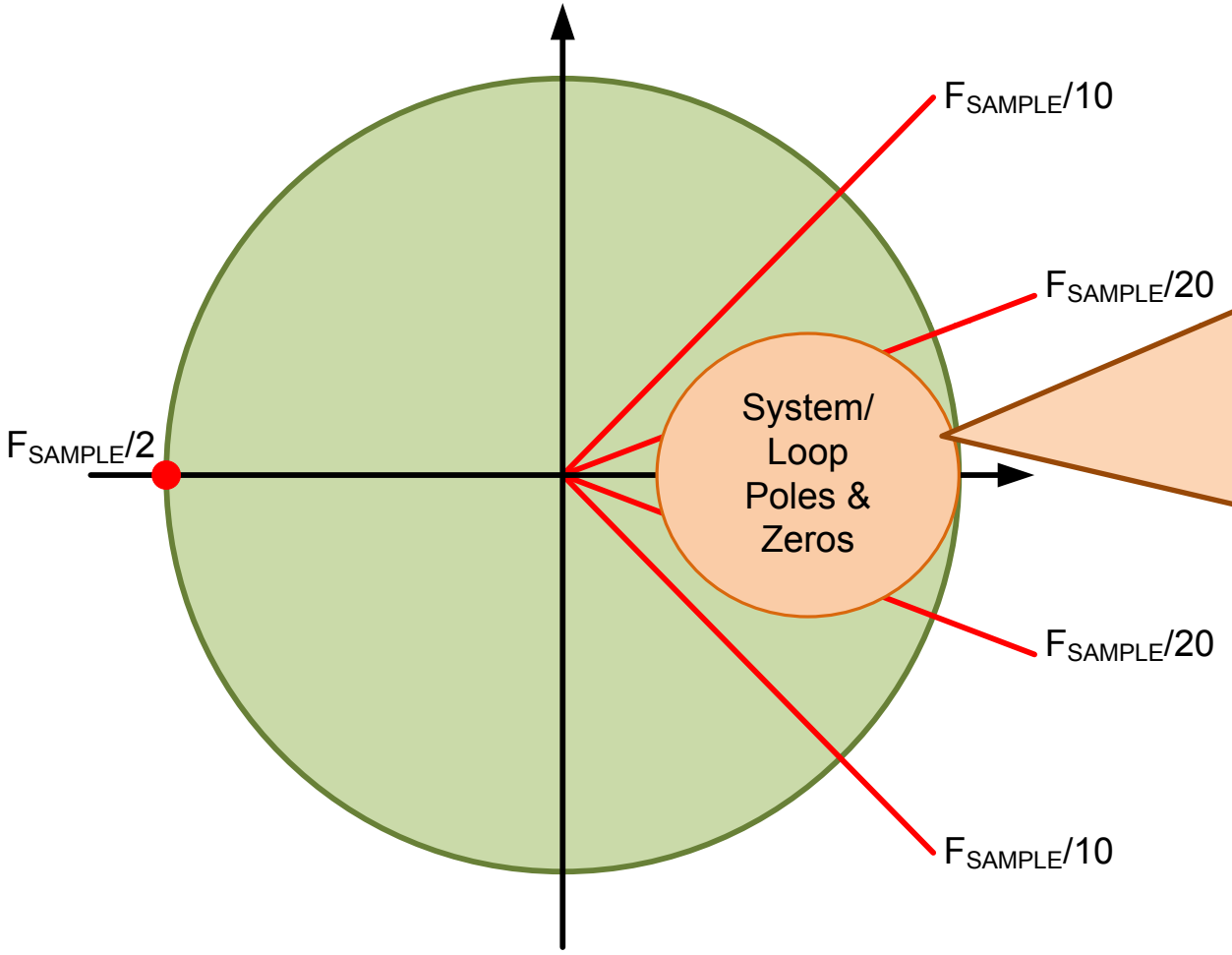
Undersampling



Oversampling



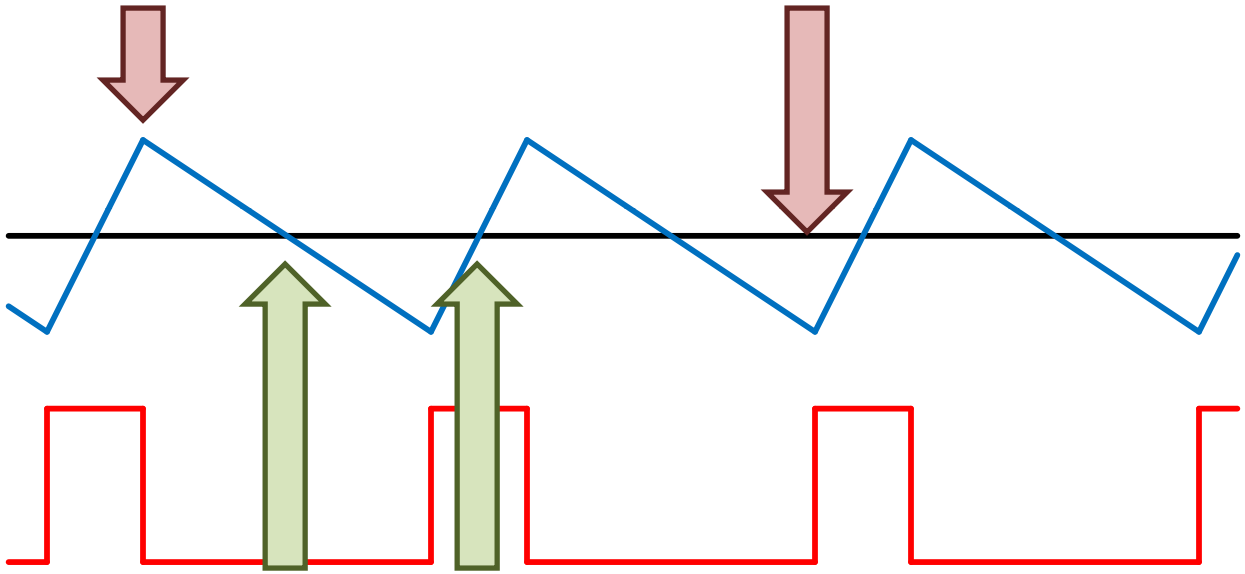
Oversampling



Close spacing of poles & zeros means high precision arithmetic!

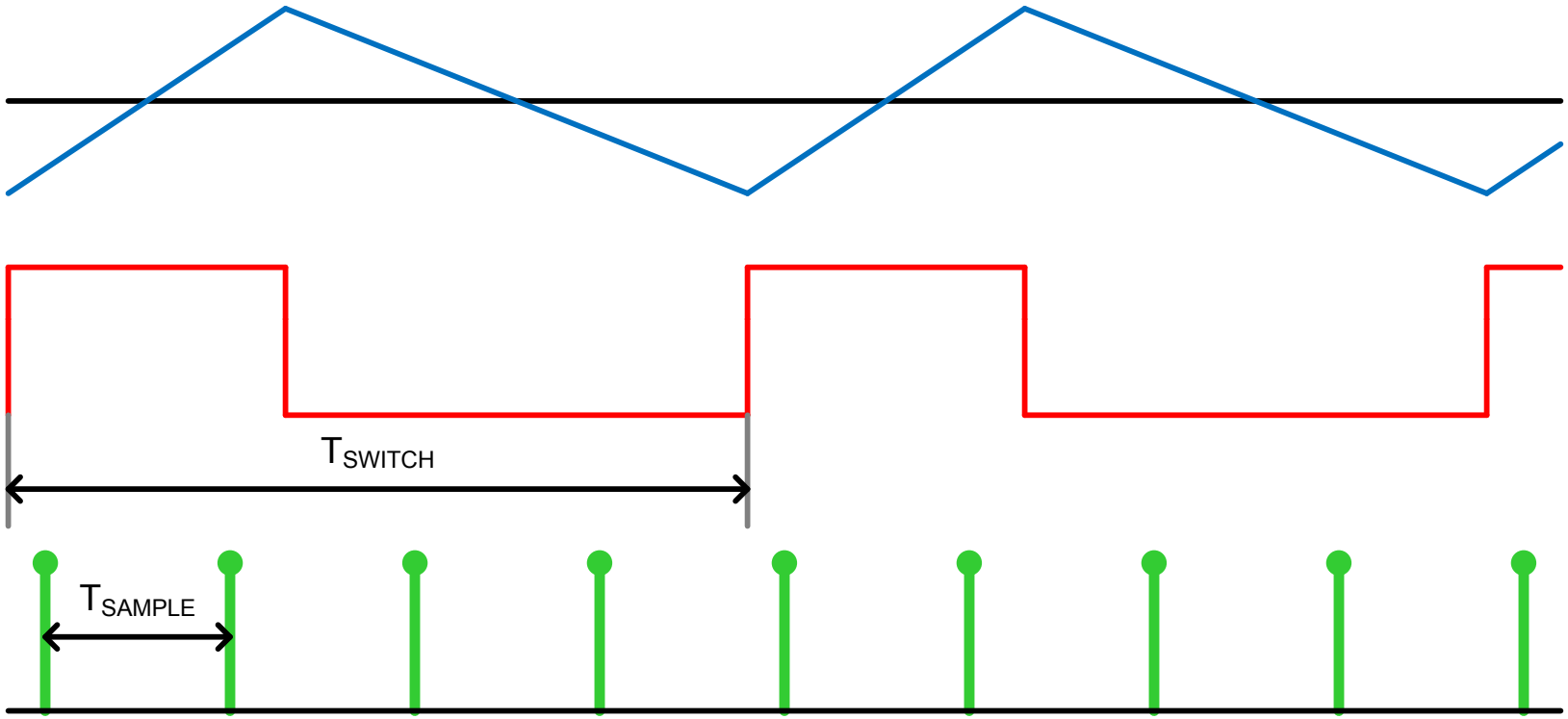
$F_{SAMPLE} = F_{SWITCH}$

Do not sample at switching points – Too much noise!



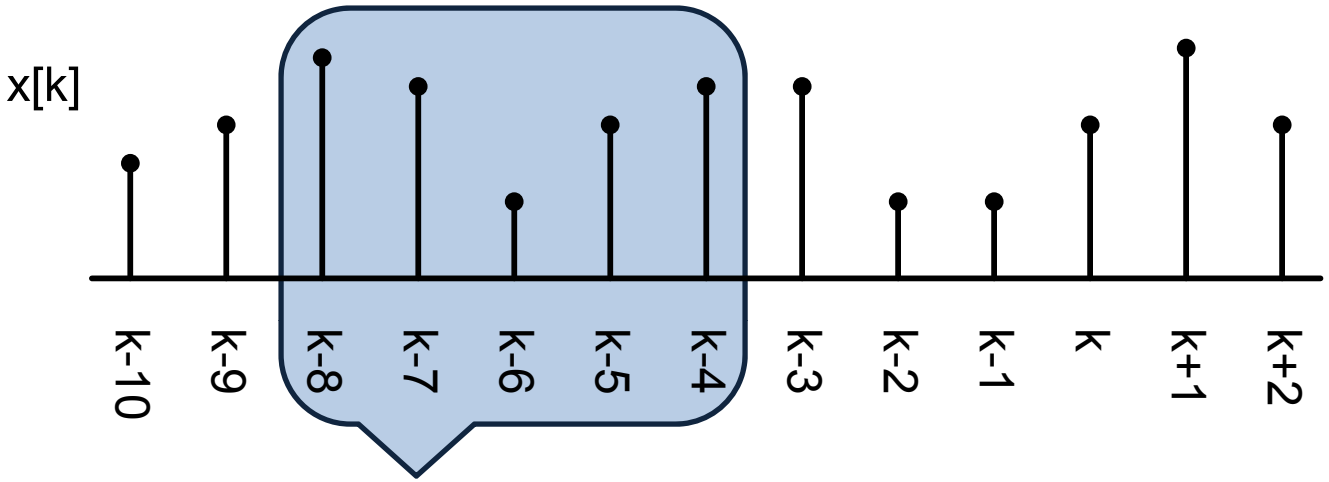
Sample at midpoint of on or off time
Quiet, Gives average value

Oversampling Revisited



If FSWITCH = N·FSAMPLE then one can a moving average filter to get the average over one cycle

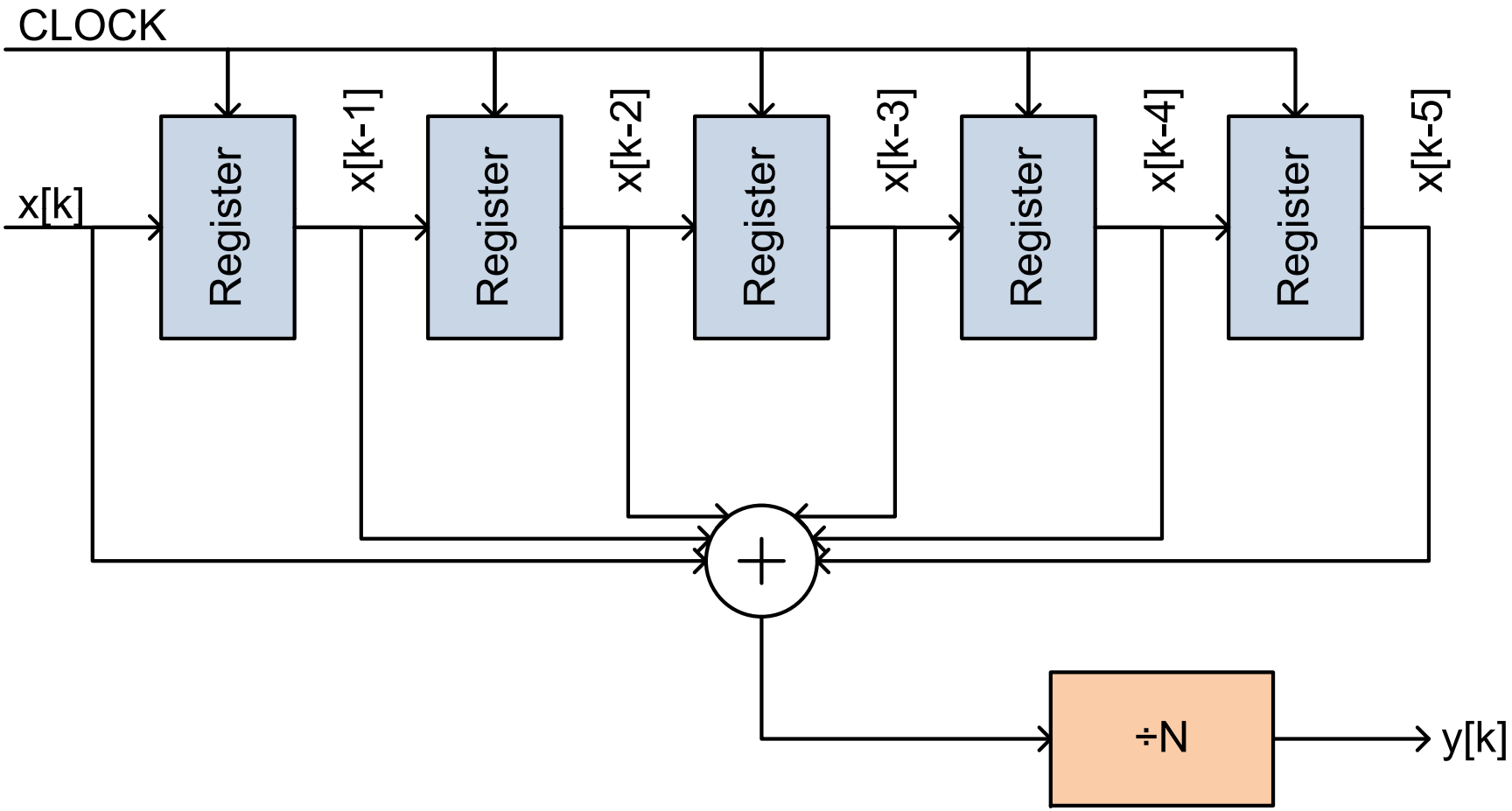
Moving Average Filter



$$y[k] = \frac{1}{N} \cdot \sum_{k-N+1}^{k} x[k] = \frac{x[k] + x[k-1] + \ldots + x[k-N+1]}{N}$$

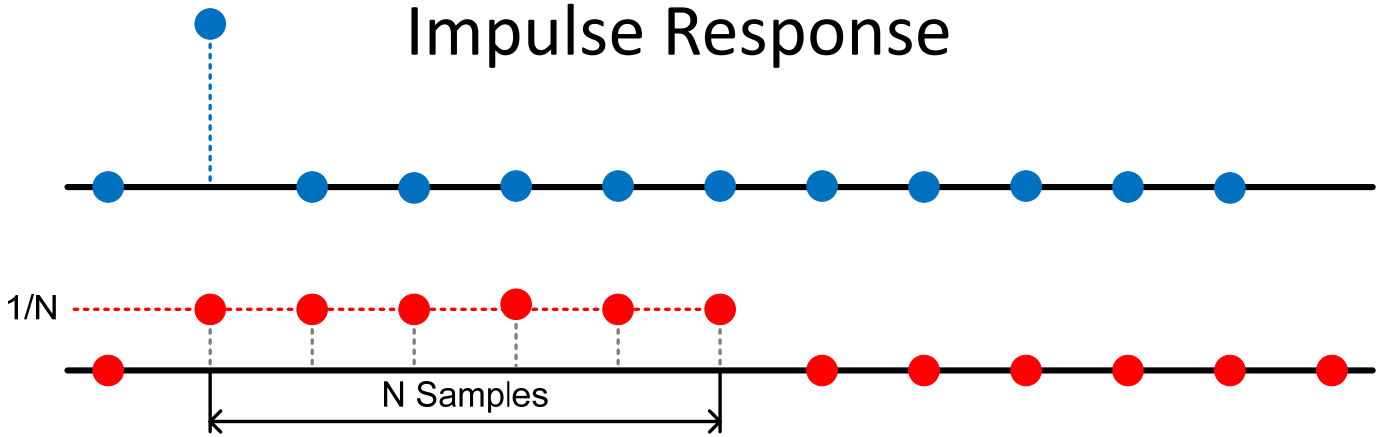
Averages the last N samples

Moving Average Filter



Moving Average Filter

Impulse Response



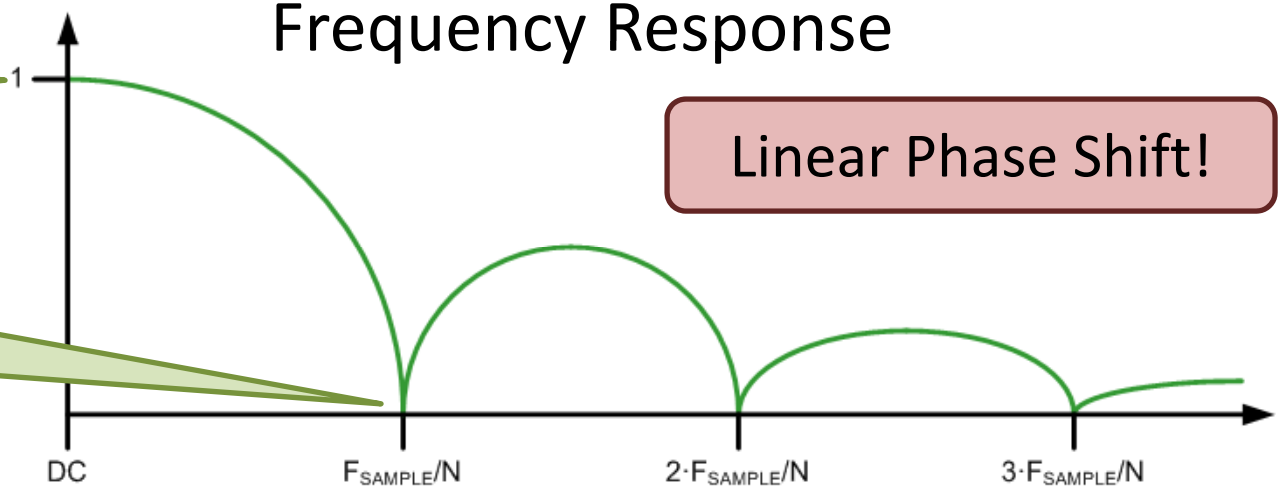
"Finite Impulse Response (FIR)" Filter

Frequency Response

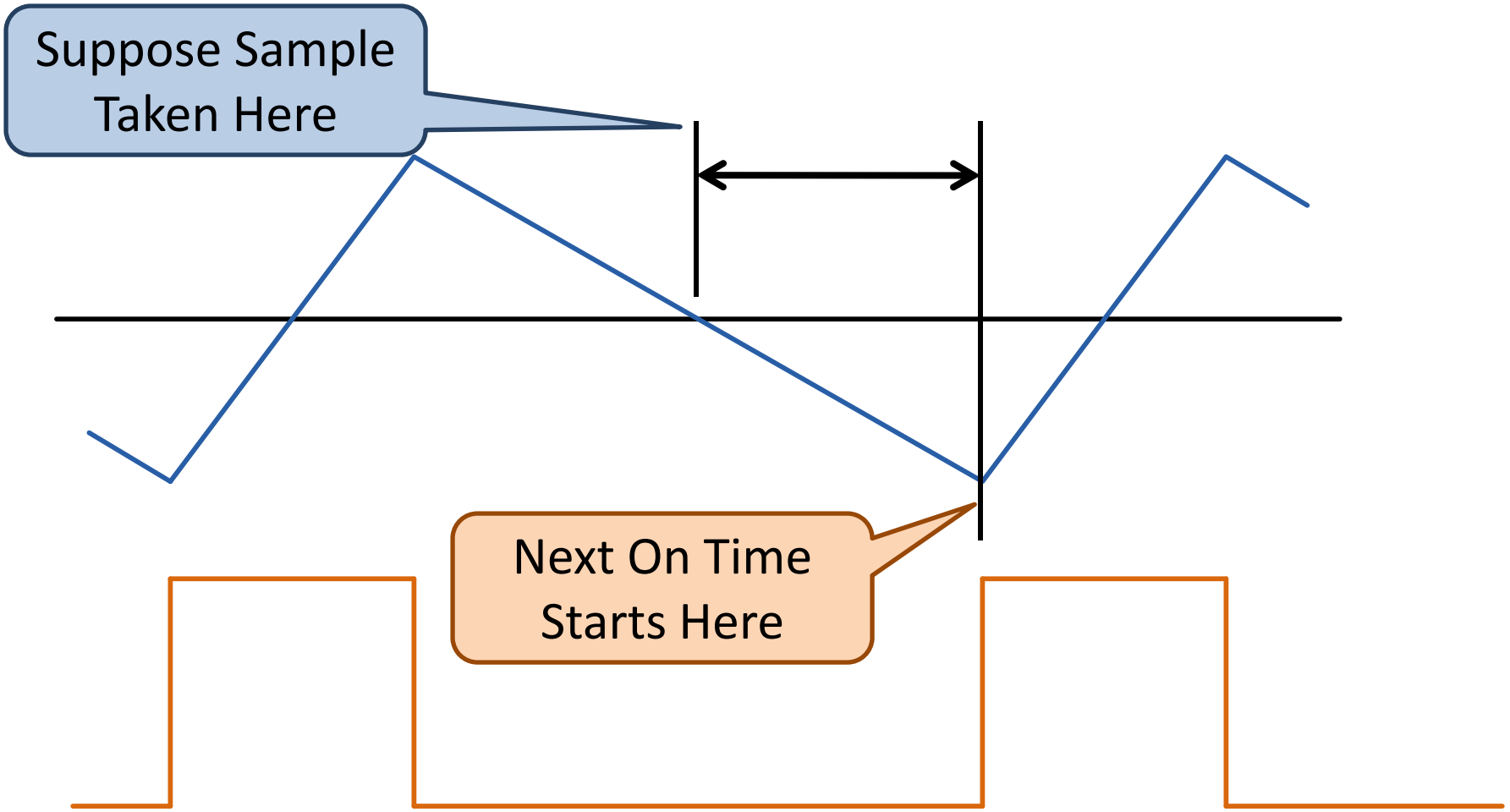
DC Gain = 1

Zeros At F_{SAMPLE}/N And Multiples

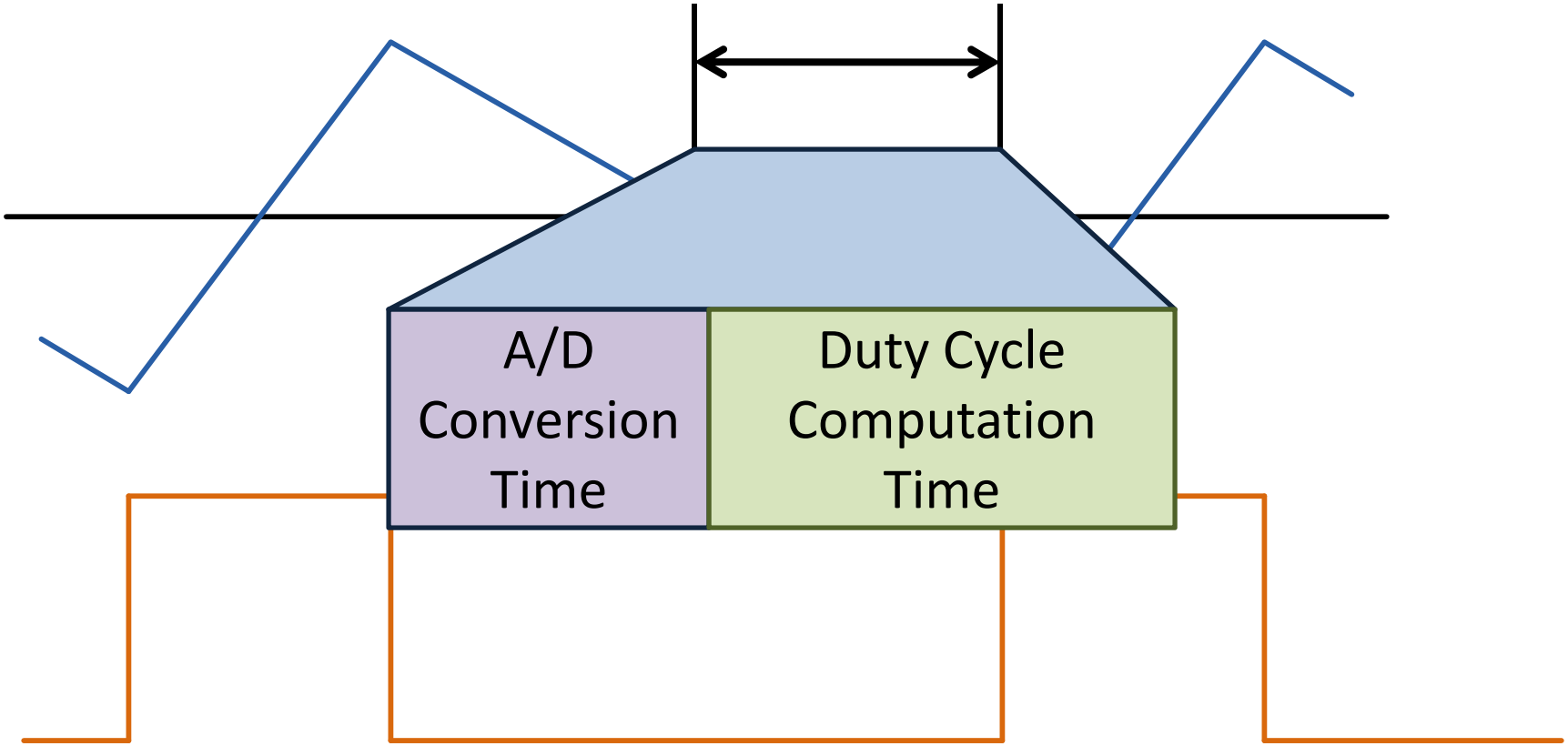
Linear Phase Shift!



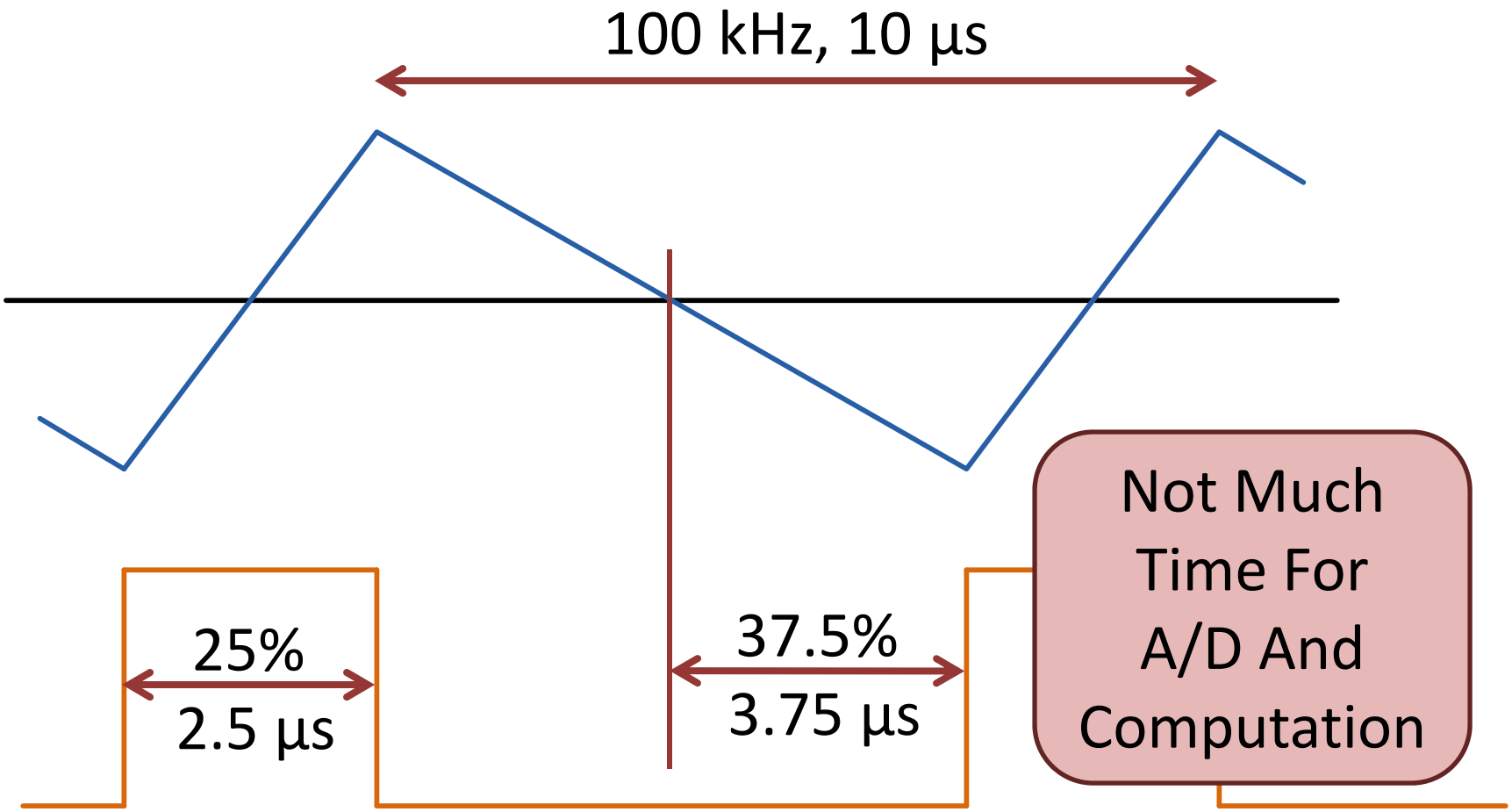
Circuit Speed Issue



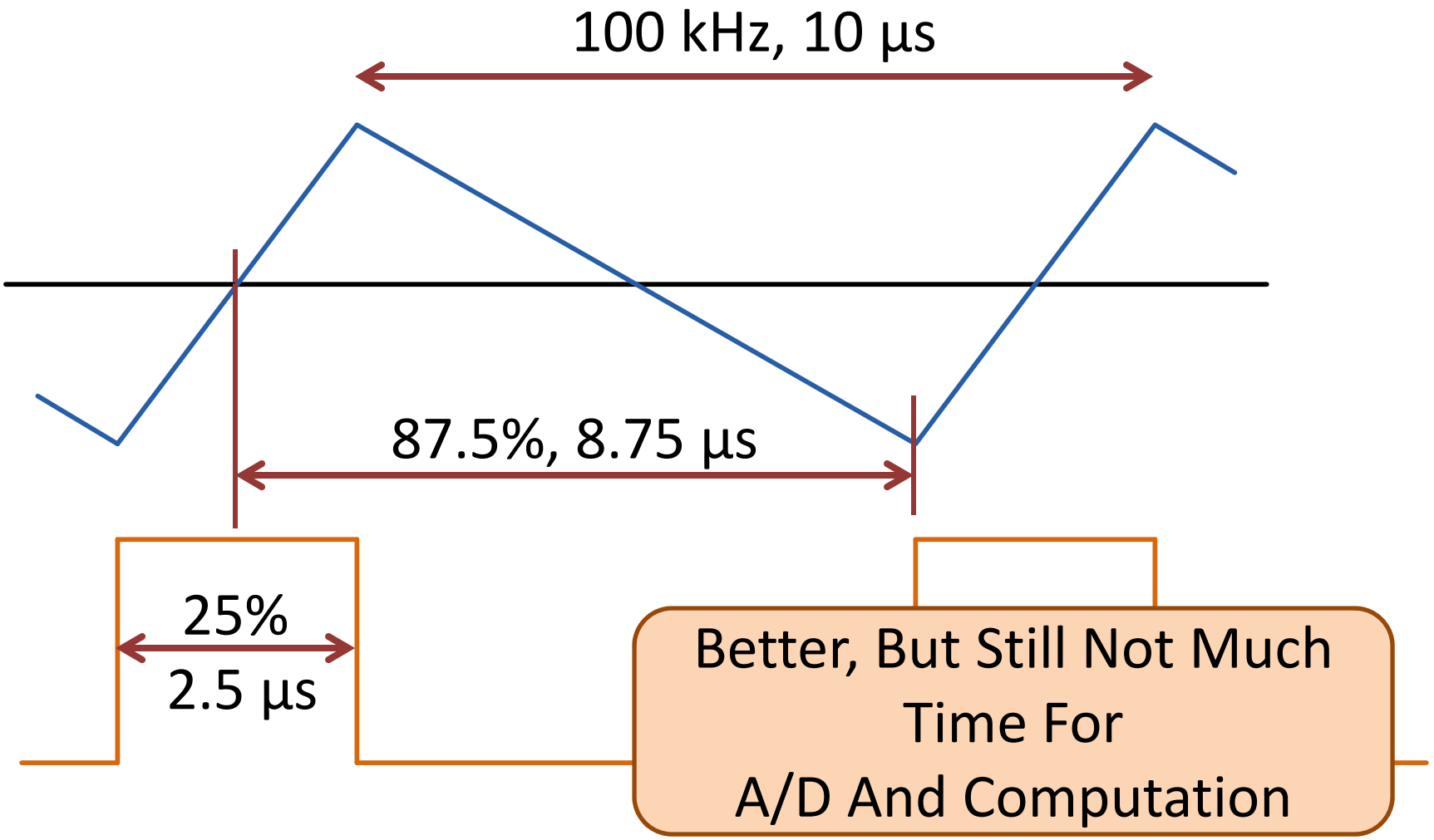
Circuit Speed Issue



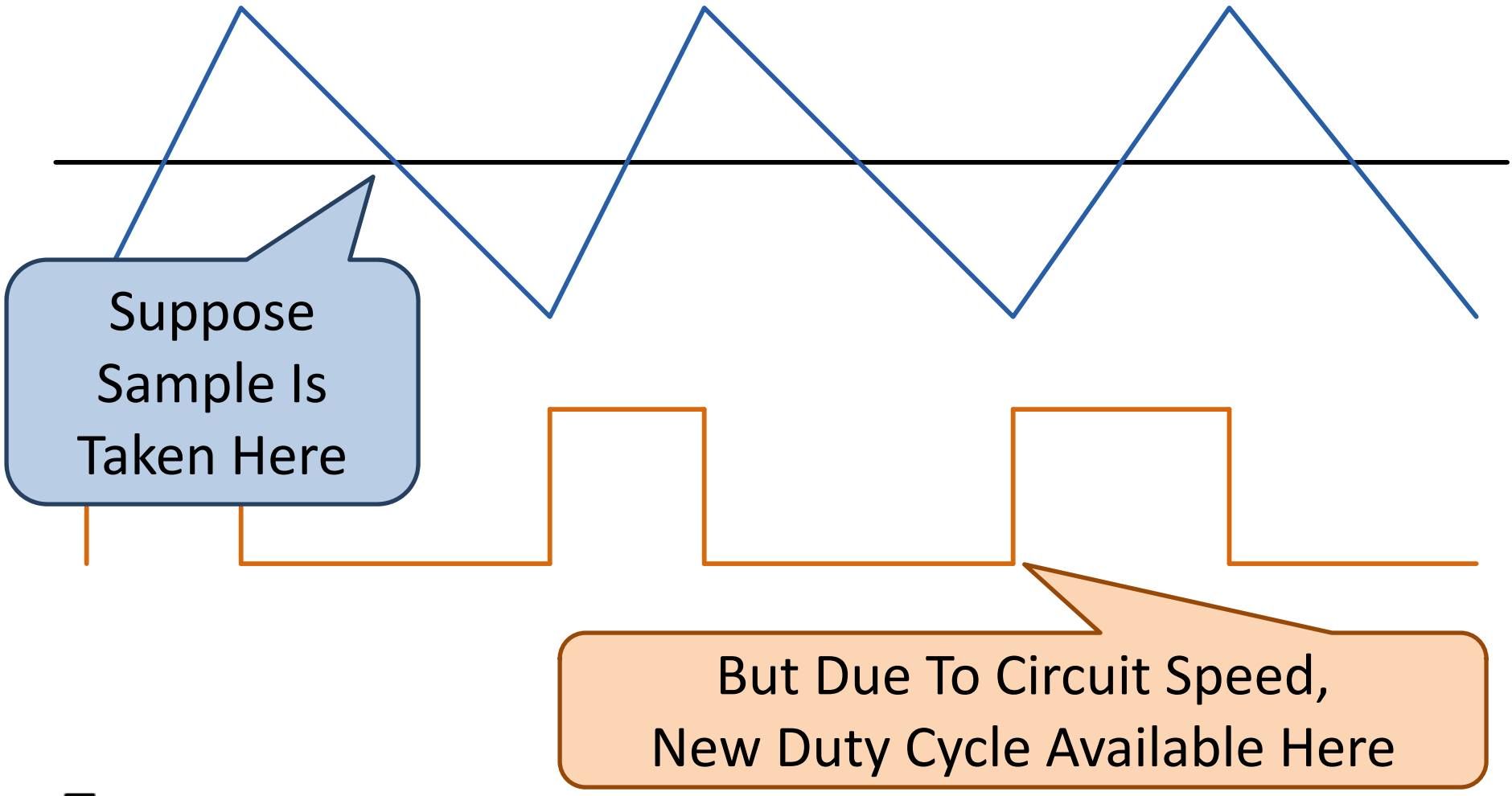
Circuit Speed Issue



Circuit Speed Issue

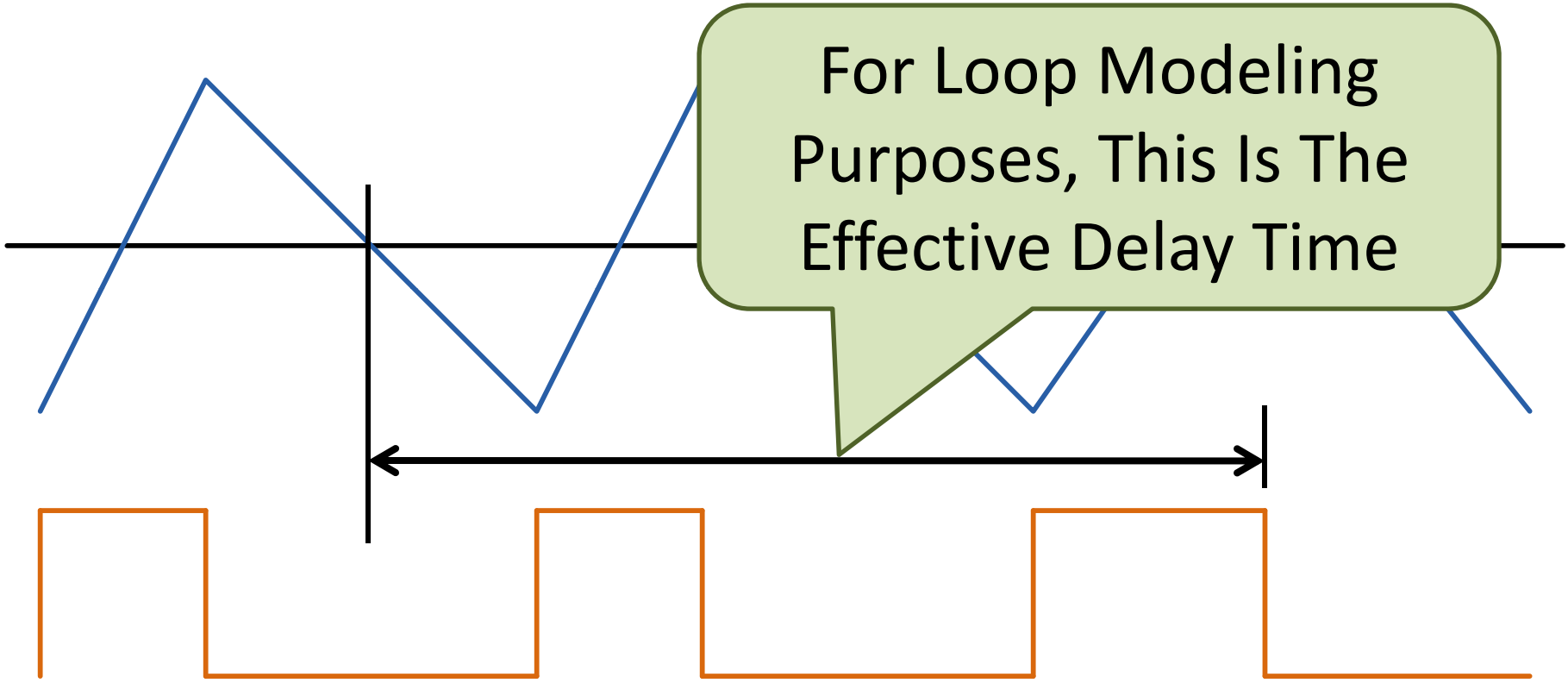


Delay Time = Loop Phase Shift



EMBEDDED POWER LABS

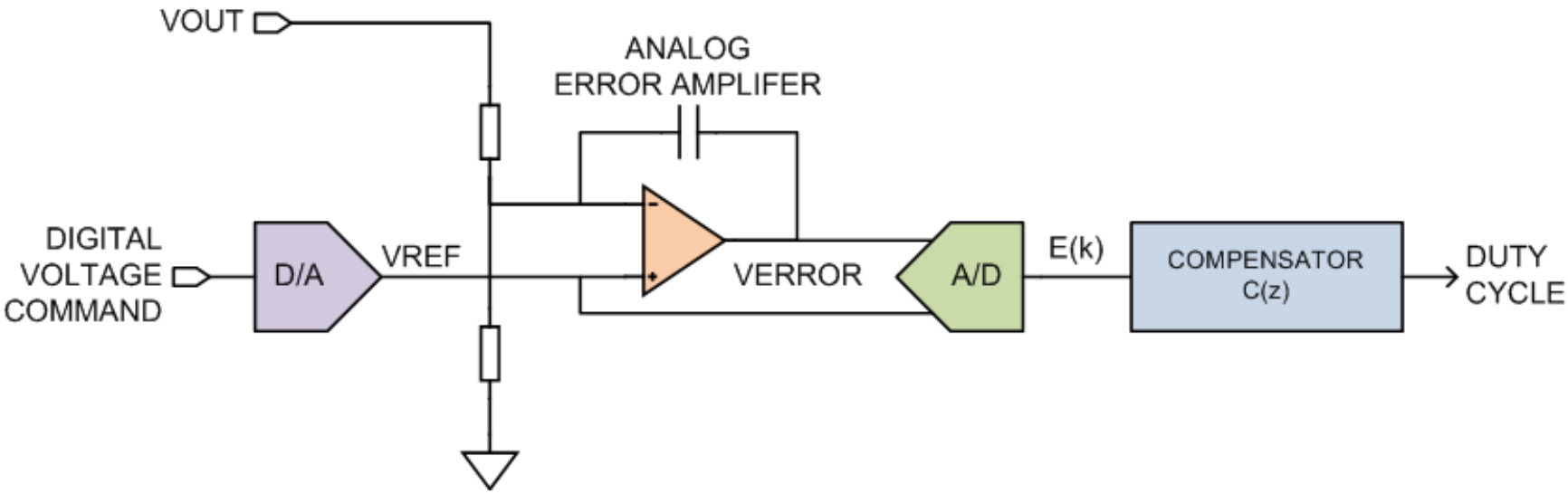
Delay Time = Loop Phase Shift



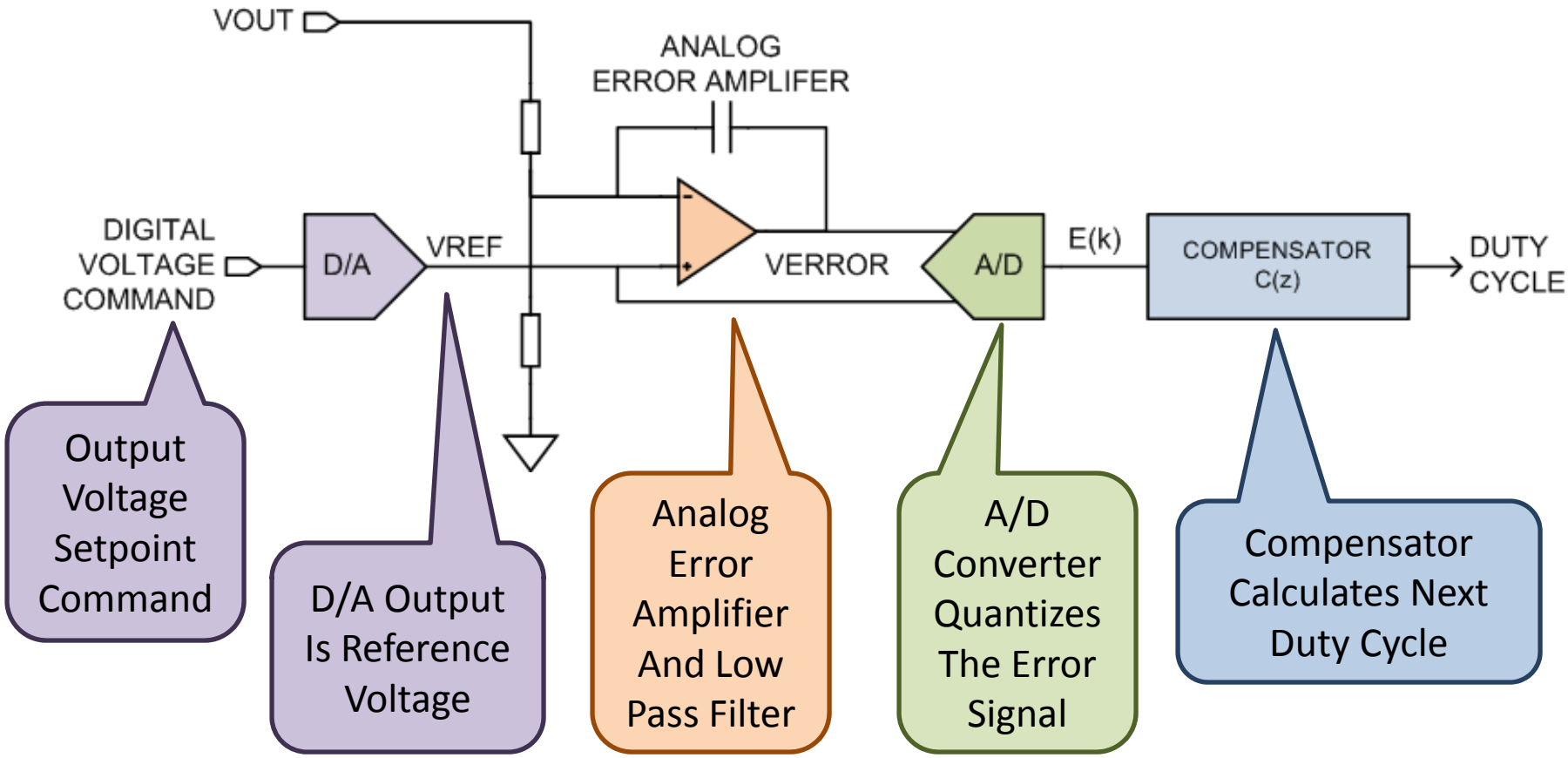
Sampling Resolution

- Not As Big Of An Issue As It Used To Be
- 10 Bit Resolution Gives 0.1% Resolution
- 10 Bits Is Fairly Economical Today
- Issues
  - ENOB In Noisy Environment?
  - Need Accurate Reference
  - Need Fast Conversion Time
  - Wide Range Output: Resolution At Lowest Voltage Drives The Number Of Bits

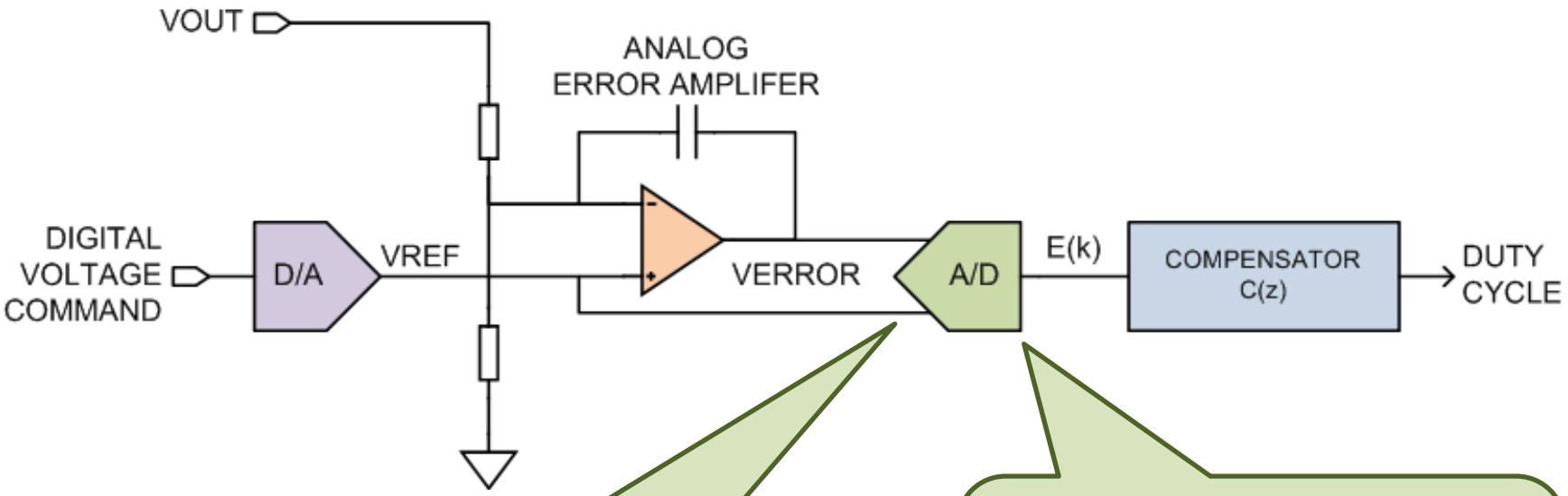
Analog Error Amplifier Approach



Analog Error Amplifier Approach



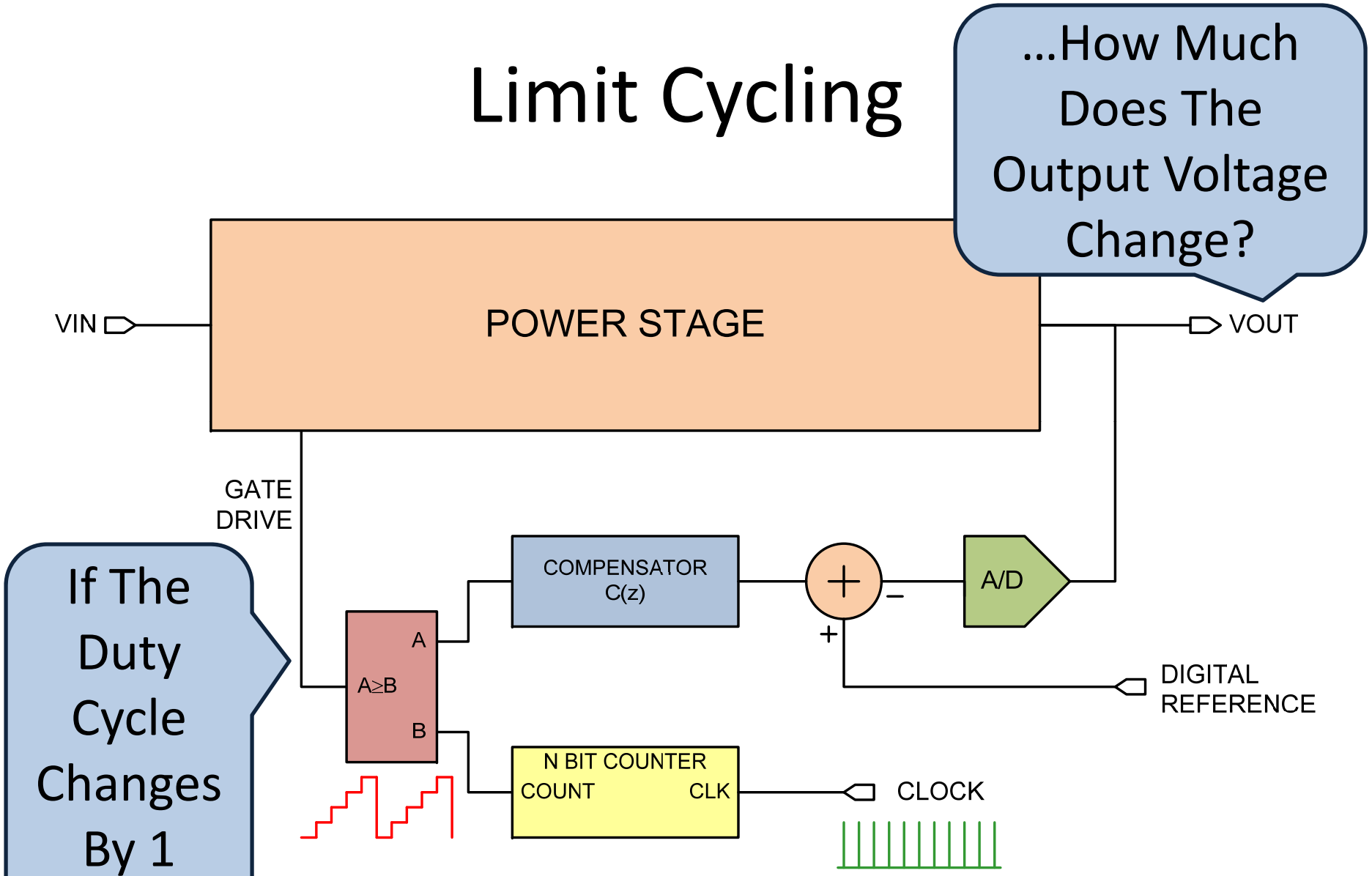
The Big Advantage



A/D Does Not Have To Have High Resolution!

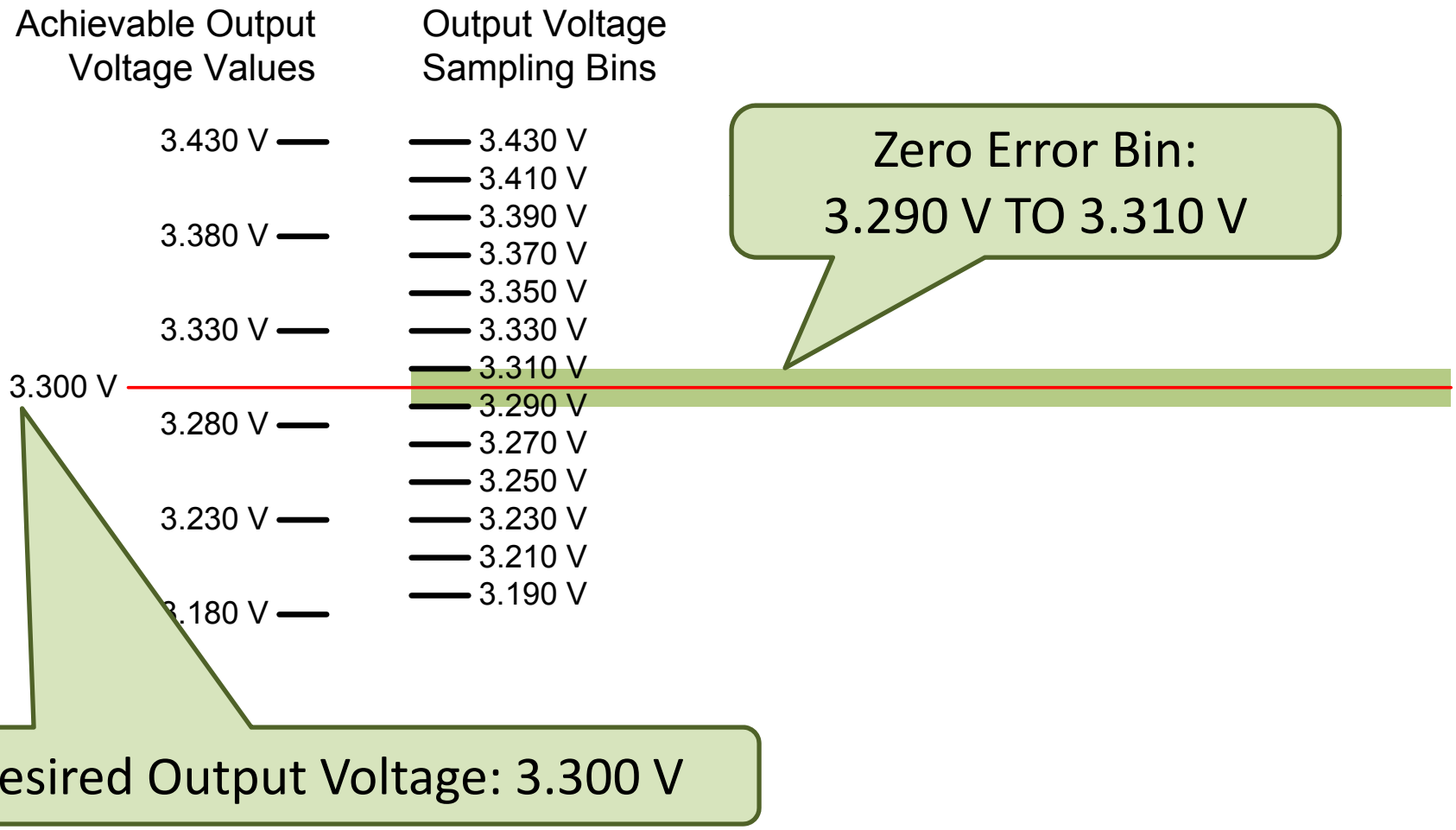
Non-Linear Quantization Can Have Advantages

Limit Cycling



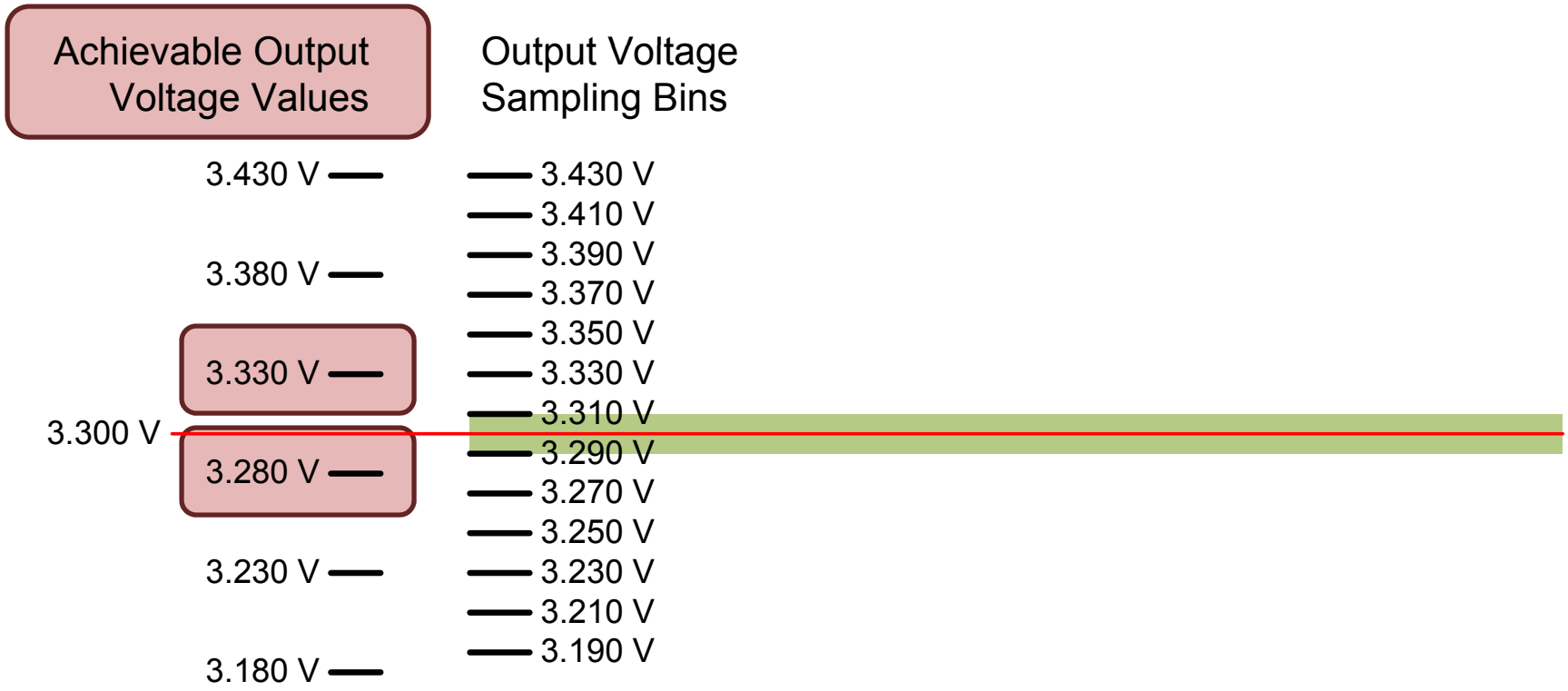
EMBEDDED POWER LABS

Limit Cycling Illustrated

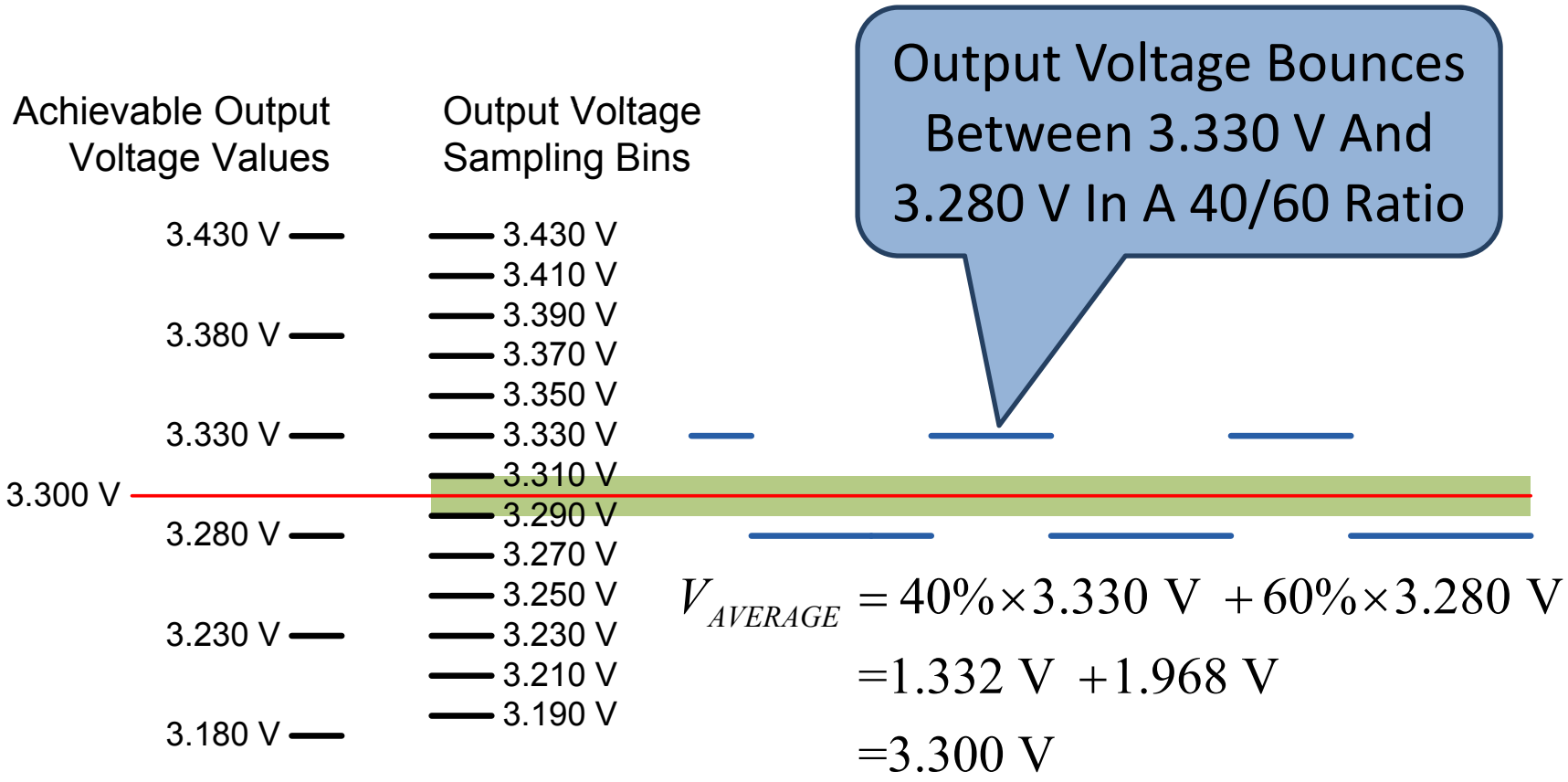


EMBEDDED POWER LABS

Limit Cycling Illustrated



Limit Cycling Illustrated



Actual Behavior Is Generally Chaotic

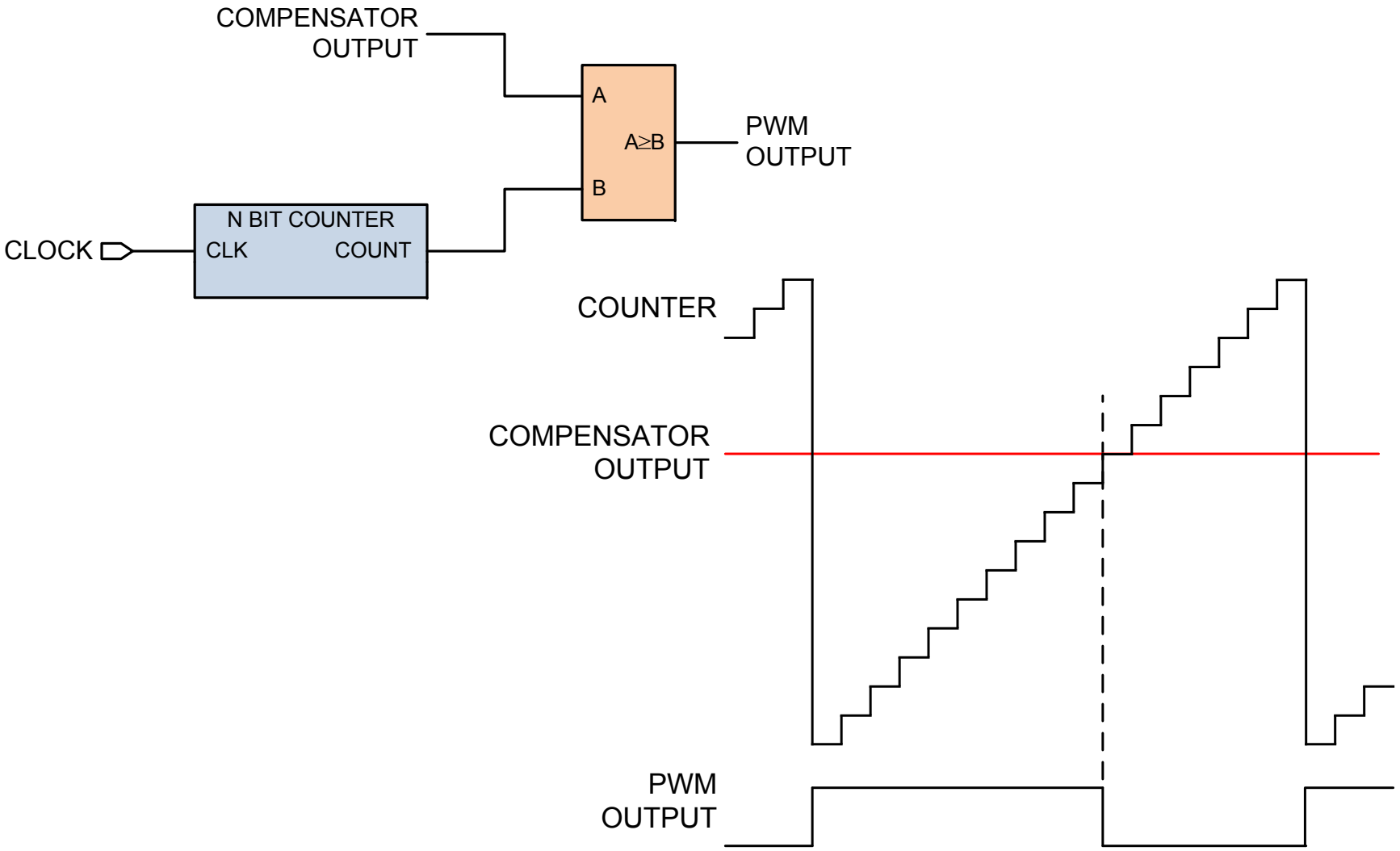
Limit Cycling

- A Real Issue
- Mainly Caused By Mismatch Between:
  - Output Sampling Resolution
  - Output Voltage Resolution
    (=DPWM Resolution × Power Stage Gain)
- Can Also Be Caused By Rounding Errors In Compensator Calculations

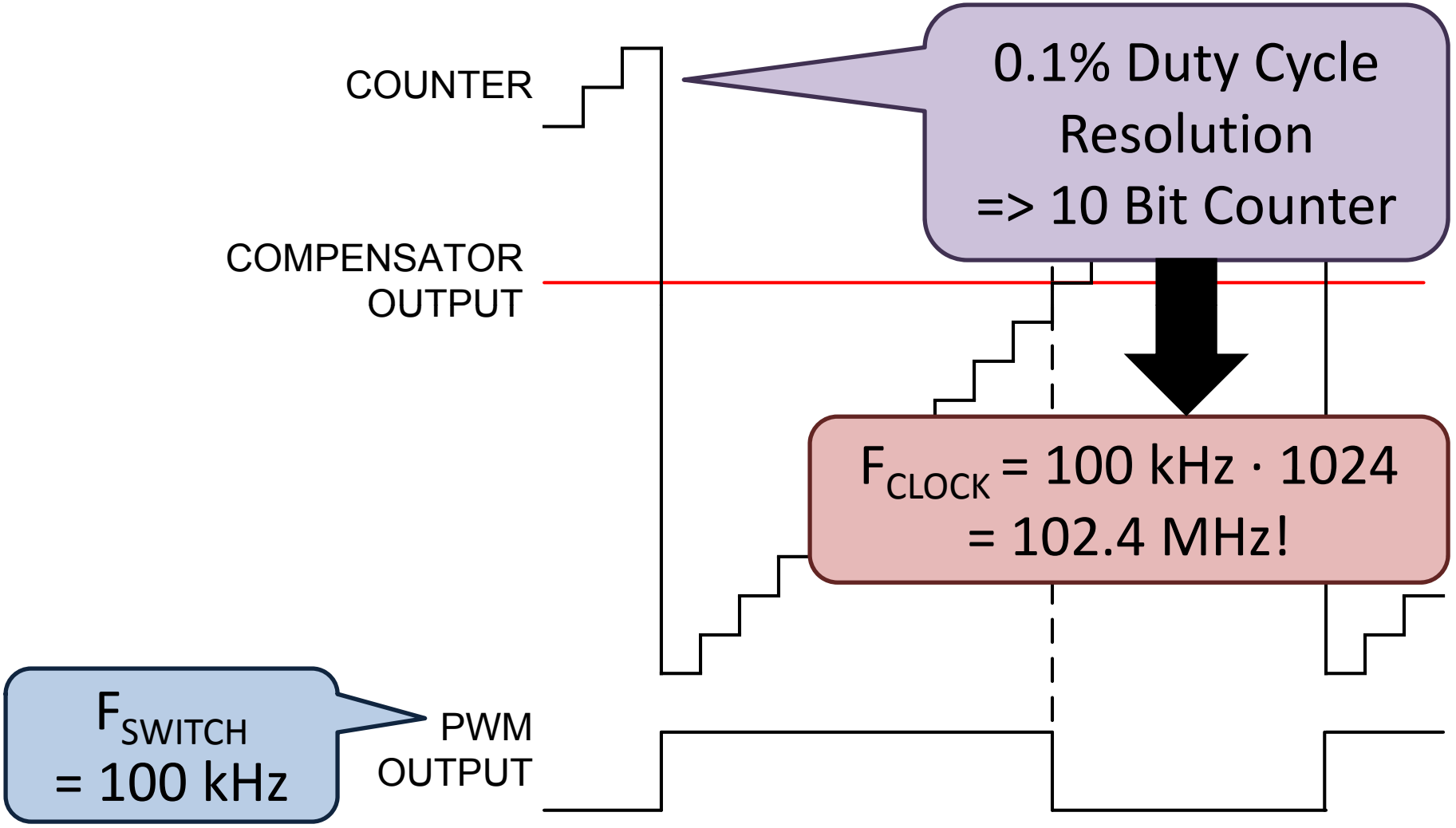
Limit Cycling And Stability

- A Power Supply That Limit Cycles Is Technically Stable
- Output Voltage Remains Bounded
- Even So, Limit Cycling Is Undesirable
  - Freaks Out The Customers
  - Creates Noise Problems

Classical Digital PWM

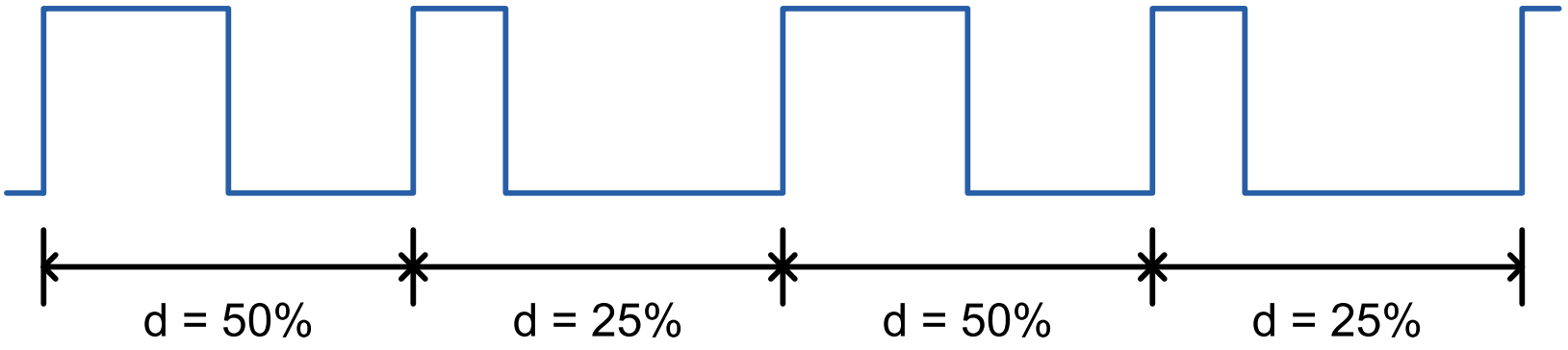


Classical Digital PWM Issue



Classical Solution: Dither

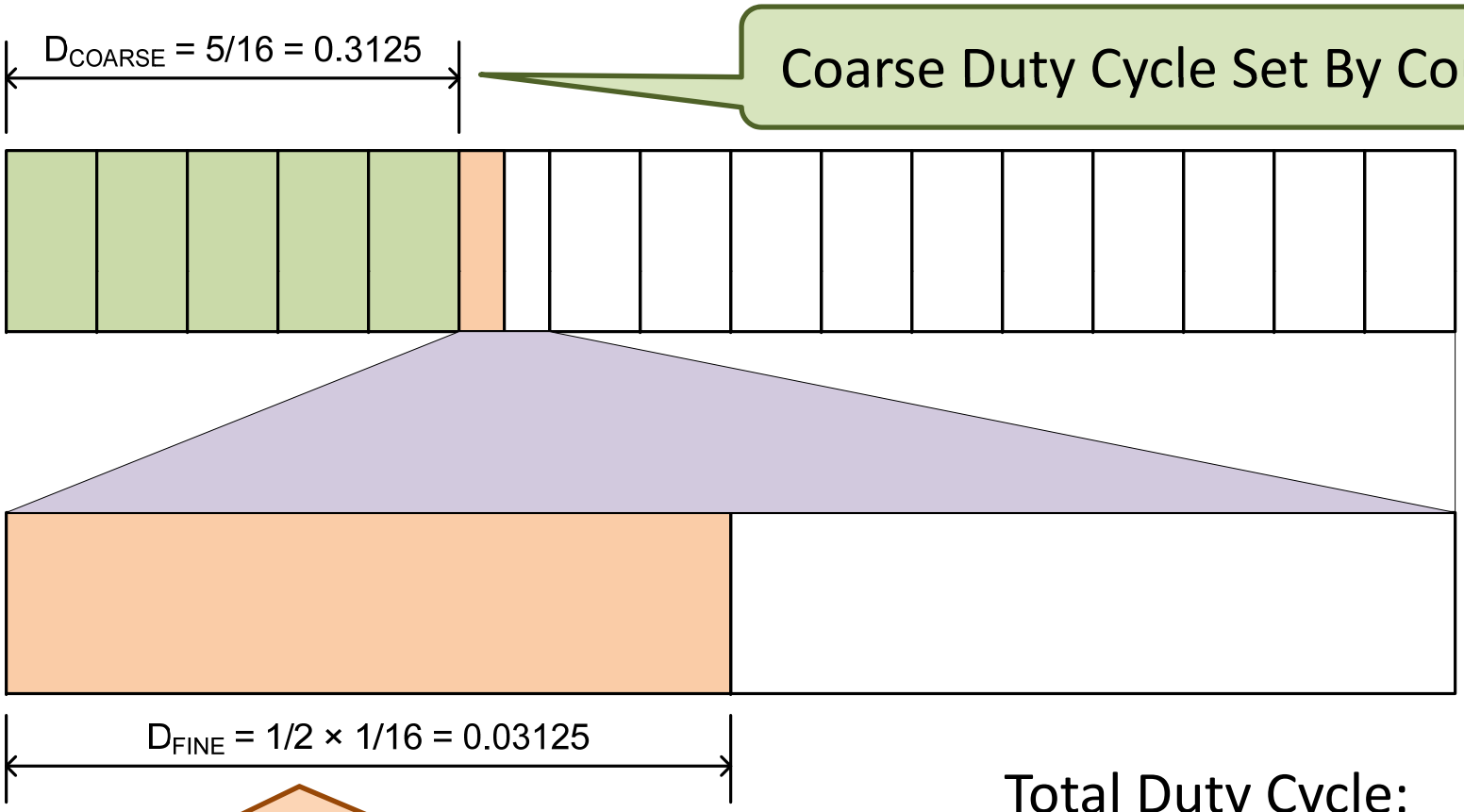
- Example: Alternating Pulses Of 50% And 25% Duty Cycle
- Average Duty Cycle: 37.5%
- Issue: Now Have $F_{SWITCH}/2$ Frequency Component In The Output



More Advanced Approaches

- Delay Lines
- Hybrid Delay Lines And Counters
- Many Possibilities To Trade Timing Resolution, Clock Speed, Power Consumption, Die Size
- See A. Syed*, et al.*, "Digital pulse width modulator architectures," in *Power Electronics Specialists Conference, 2004. PESC 04. 2004 IEEE 35th Annual*, 2004, pp. 4689-4695 Vol.6.

Hybrid DPWM Concept



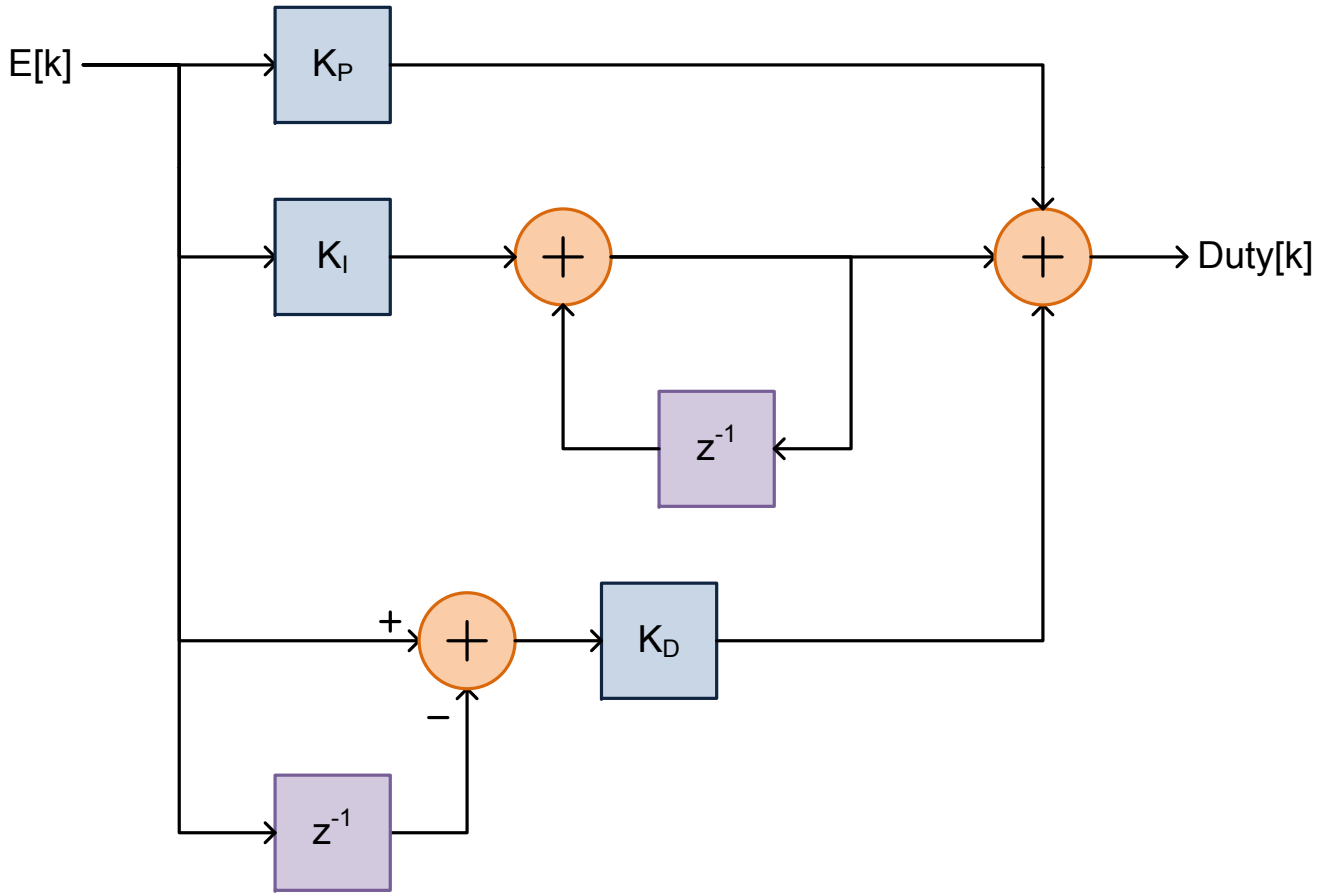
Total Duty Cycle:

0.3125 + 0.03125 = 0.34375

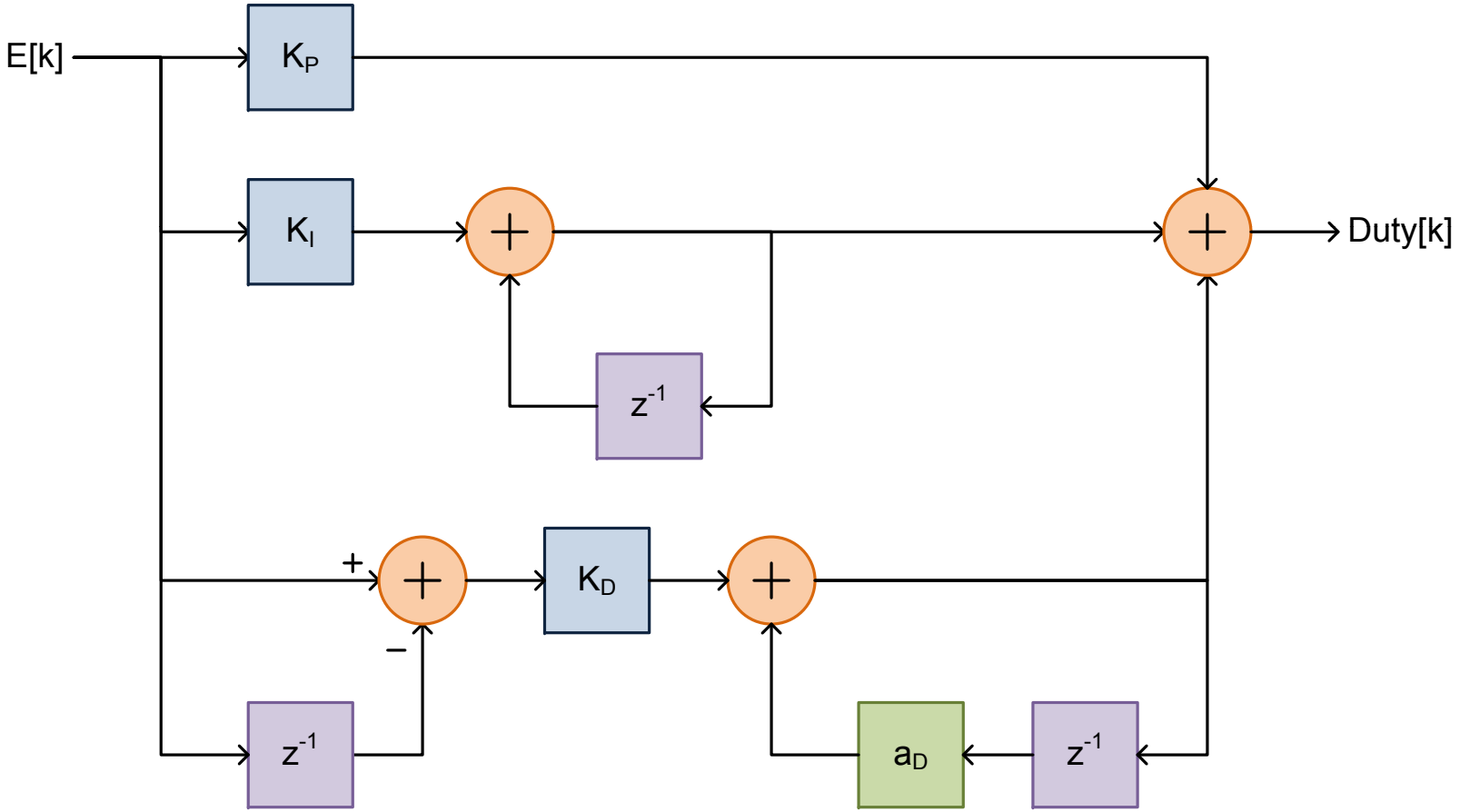
EMBEDDED POWER LABS

Compensator: PID

$$Duty[k] = K_P \cdot E[k] + K_I \cdot \sum E[k] + K_D \cdot \left(E[k] - E[k-1]\right)$$

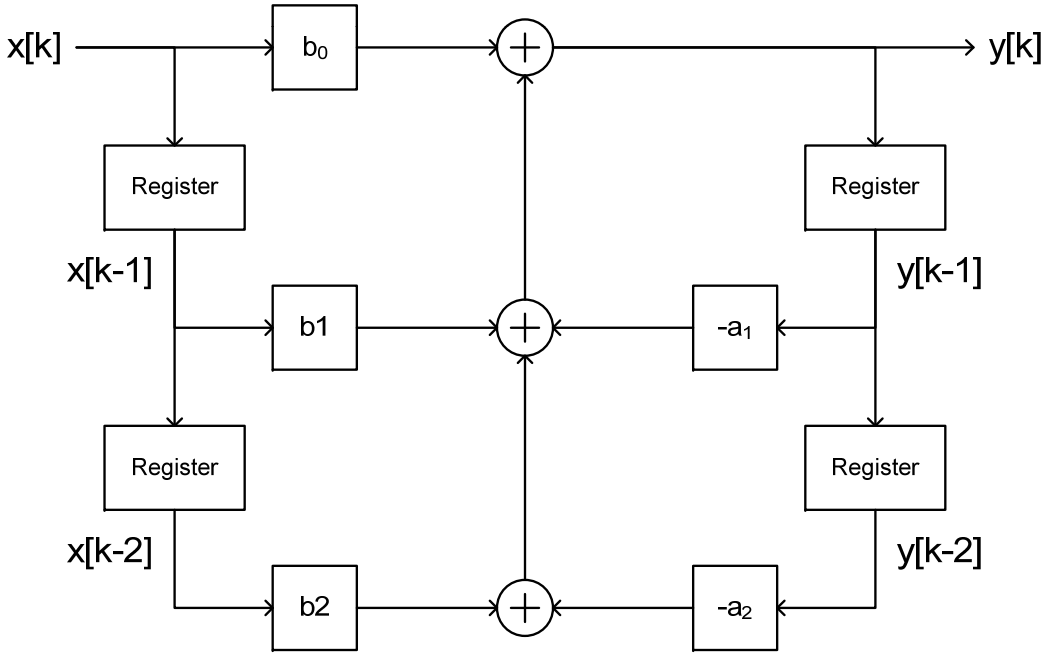


Compensator: PID



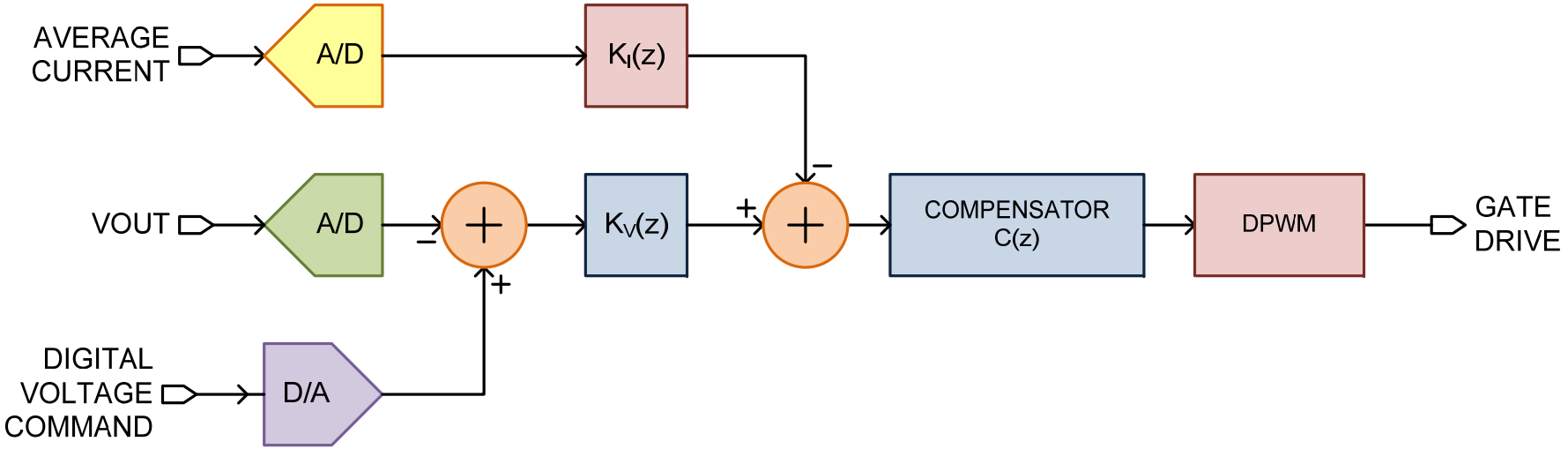
Compensator: Frequency Domain

- Pole And Zero Placement As In Continuous Time
  - Lead/Lag
  - Type 2, Type 3

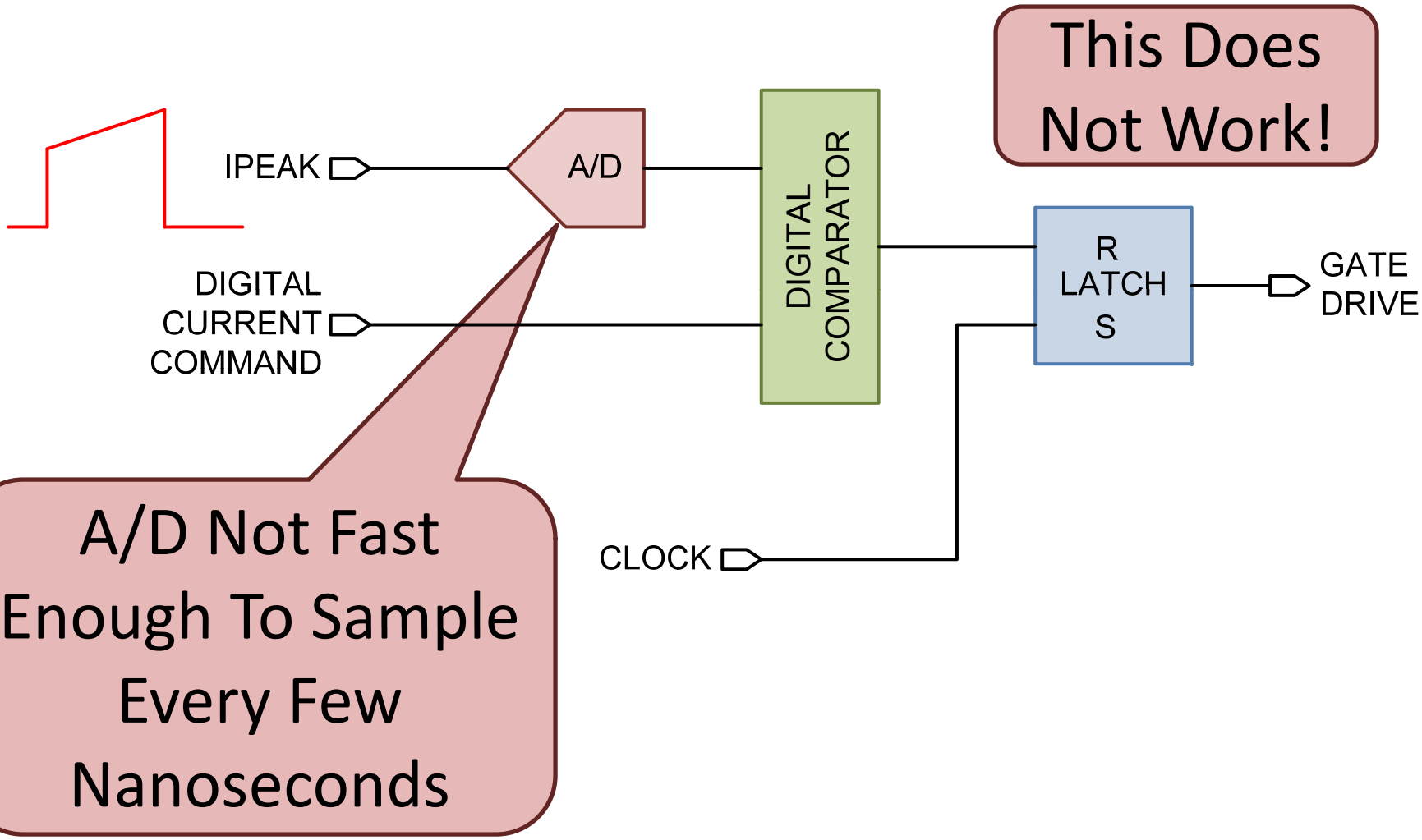


Digital Average Current Mode Control

- Digitize The Analog Version
- Sense Average Current The Same Way As Analog
- Similar Speed And Resolution Issues

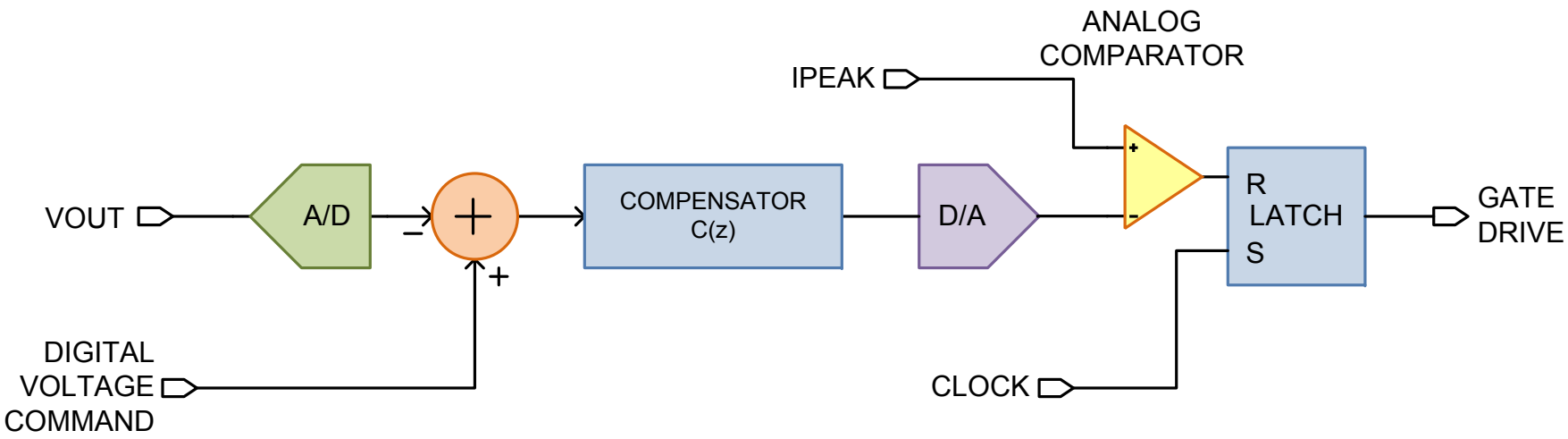


Peak Digital Current Mode Control



Peak Digital Current Mode Control

- Use A Hybrid Approach
  - Digital Compensator
  - Use D/A To Set Current Threshold
  - Analog Comparator To Detect The Peak



Today's Platforms

- Digital Signal Controllers Tuned For Power Supply Applications
  - Microchip Technology dsPICs
  - TI Piccolo
- Limitations
  - Single Core Processing
    - Little Opportunity For Parallel Operations
  - Instruction Speed

Today's Platforms

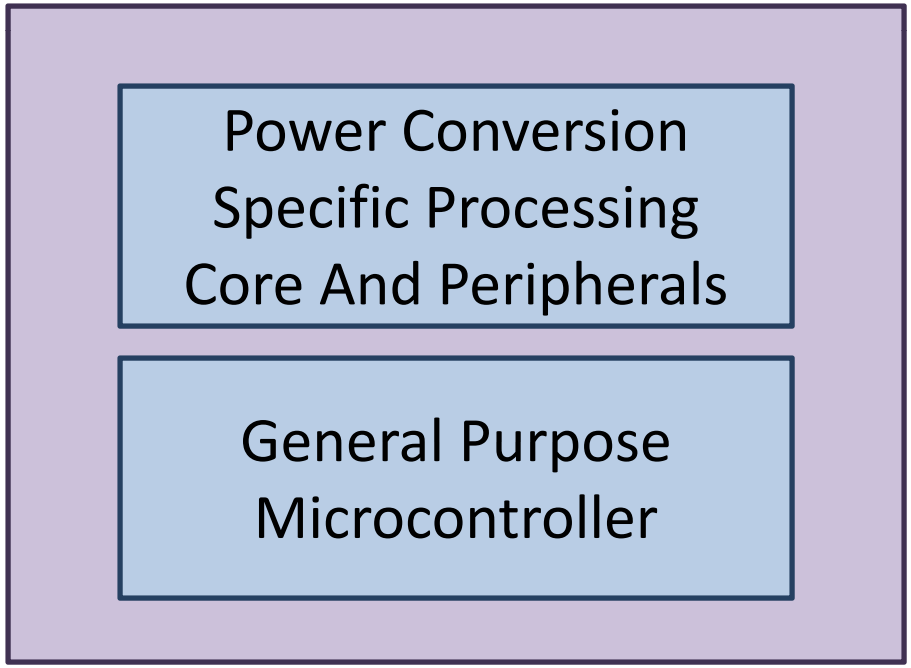
- Full ASIC Designs
  - Intersil/Zilker
  - Volterra
  - Chil
  - Maxim
  - ZMDI
  - Etc.

Key Advantage: Parallel Operations As Needed

Key Disadvantage: Little Or No Customization

Today's Platforms

- Hybrid Designs
  - Example: TI Fusion Controllers



Today's Platforms

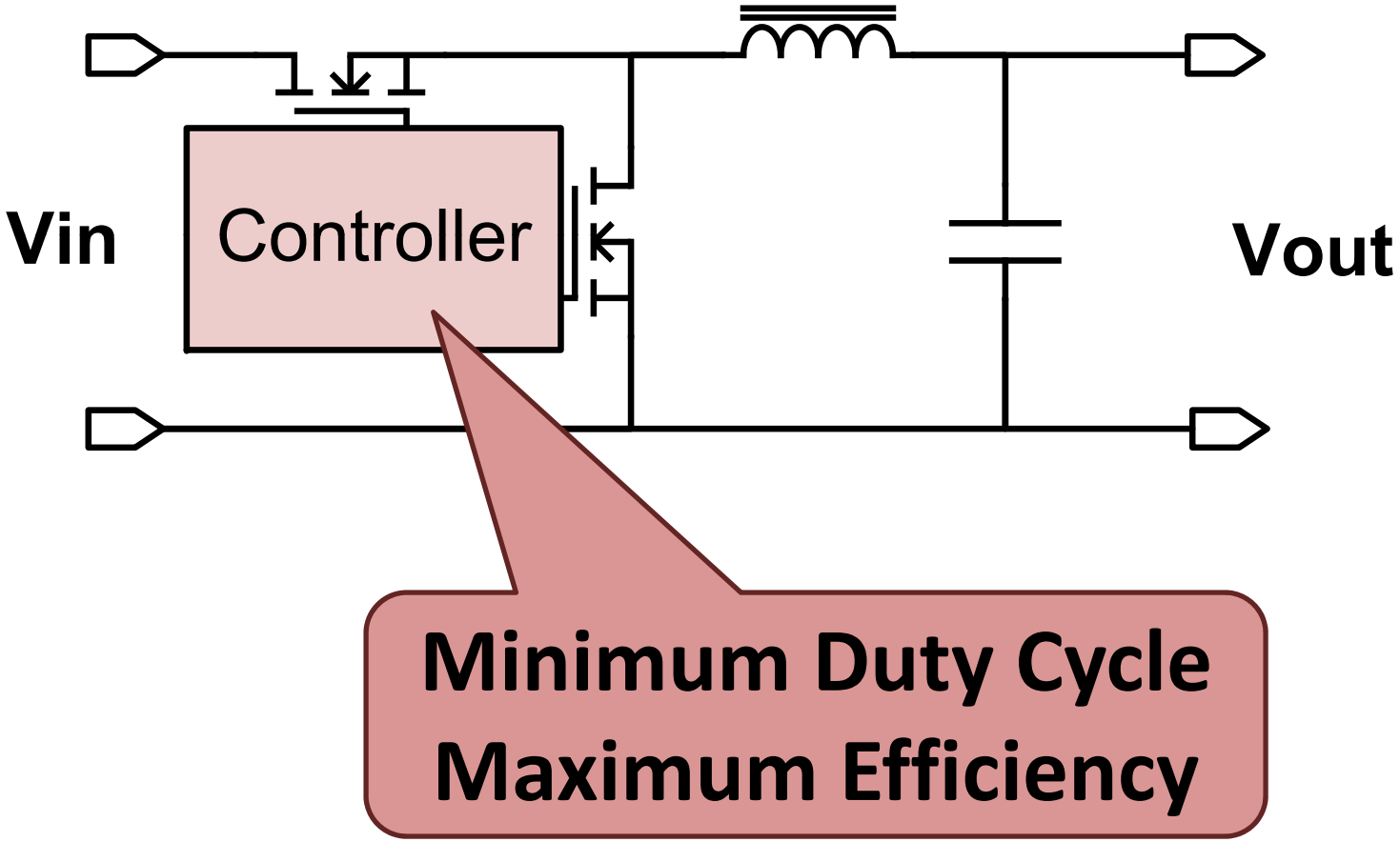
- FPGA Plus…
  - Power Supply Oriented Peripherals
  - Hard Core Microcontrollers
  - Example: Microsemi/Actel Products
- Cypress PSoC
  - Microcontroller Core
  - Configurable Analog Peripheral Blocks

Digital Control Enabled Benefits

Dead Time Optimization

- Switching Timing Is Very Important In Synchronous Rectifiers
- Too Little Dead Time:
  Shoot Through Losses
- Too Much Dead Time:
  Excessive Body Diode Losses
- This Problem Attacked Aggressively With Both Analog And Digital Techniques

Efficiency Optimization



Sensorless Optimization

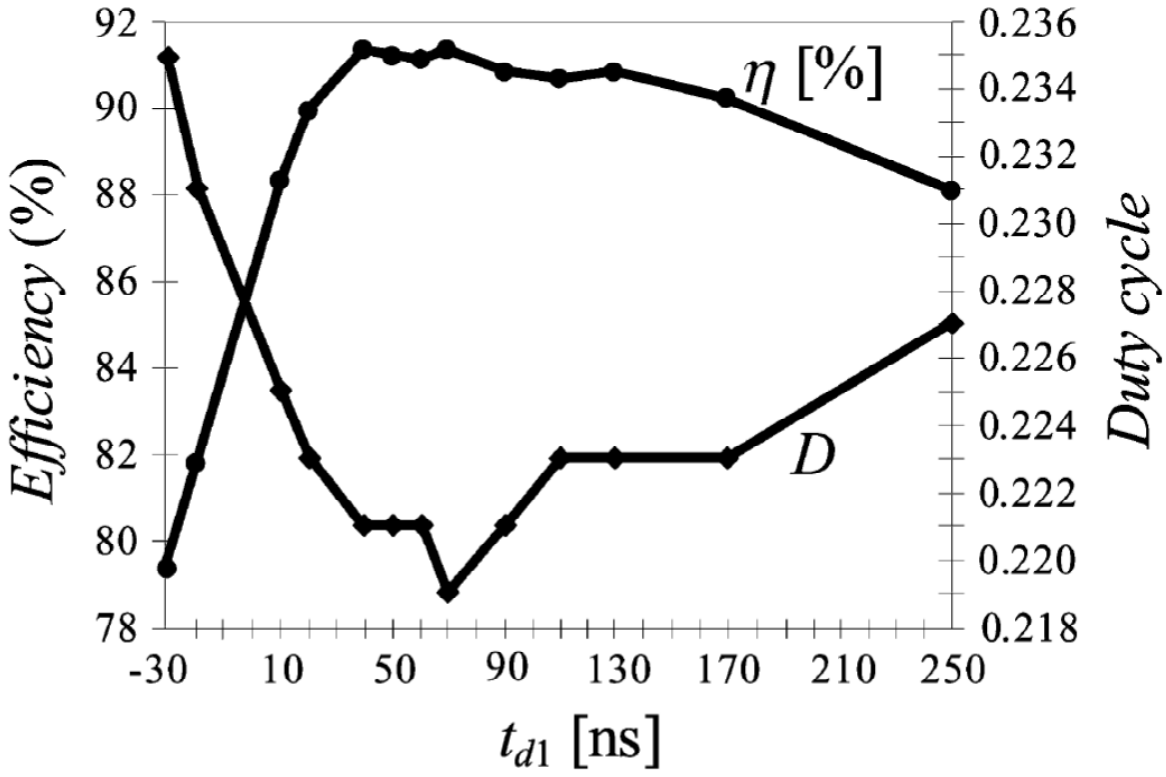
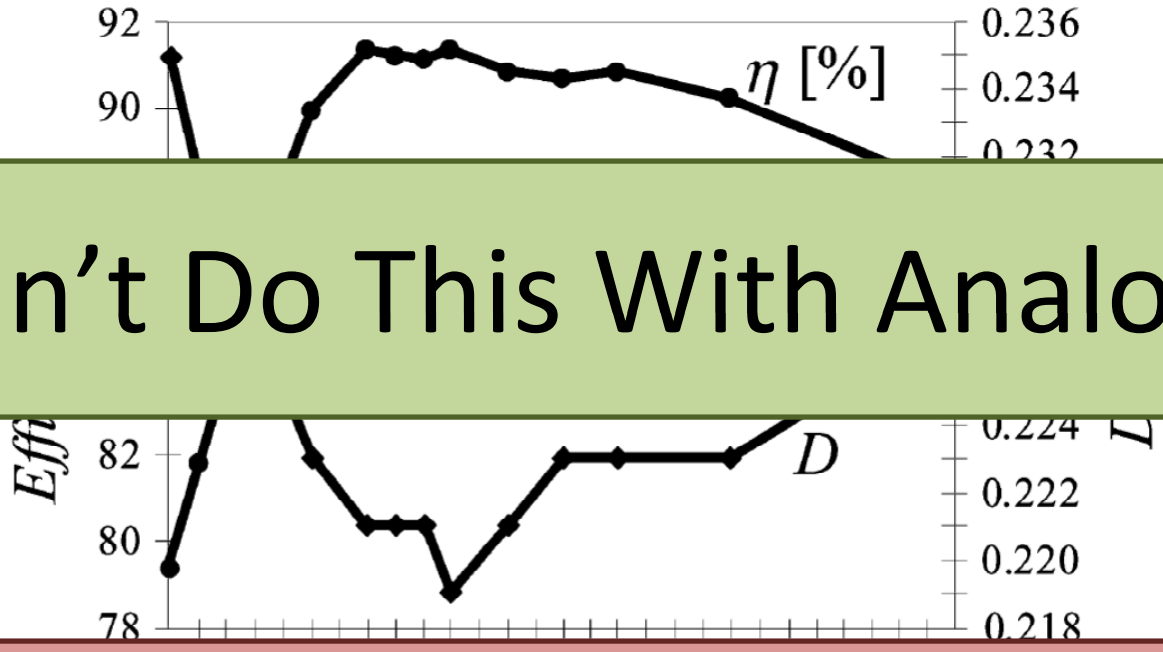


Fig. 4. Measured efficiency η and duty-cycle command D as functions of the dead-time t_{d1} in the experimental prototype.

V. Yousefzadeh and D. Maksimovic, "Sensorless optimization of dead times in DC-DC converters with synchronous rectifiers," *Power Electronics, IEEE Transactions on,* vol. 21, pp. 994-1002, 2006

Sensorless Optimization



Can't Do This With Analog!

May Be Multiple Patents On This Idea

dead-time t_{d1} in the experimental prototype.

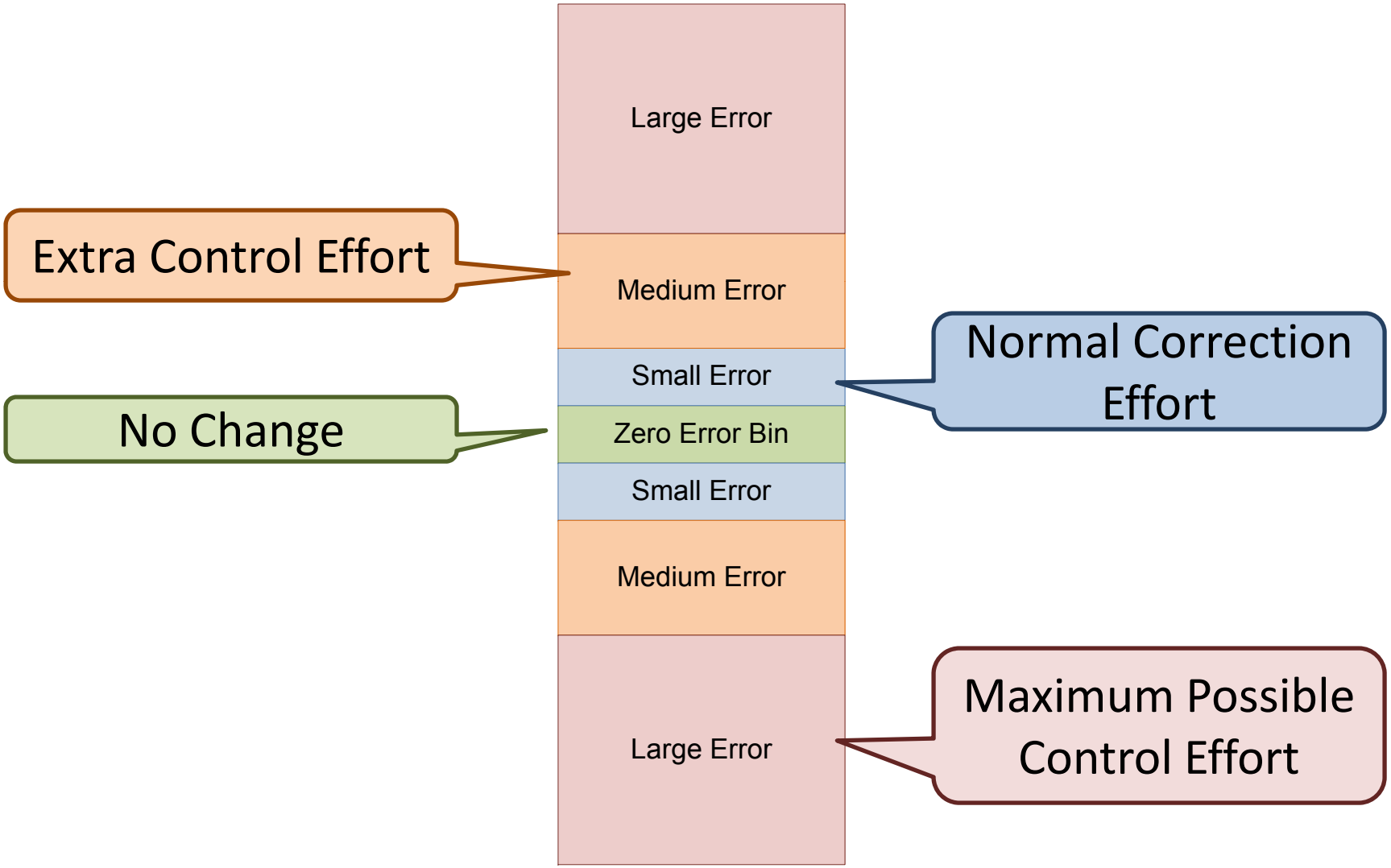
V. Yousefzadeh and D. Maksimovic, "Sensorless optimization of dead times in DC-DC converters with synchronous rectifiers," *Power Electronics, IEEE Transactions on*, vol. 21, pp. 994-1002, 2006

EMBEDDED POWER LABS

Improving Transient Response

- Two Basic Ideas
  - Nonlinear Feedback
  - Mode Switching/Gain Scheduling
- Either Can Be Implemented In Analog
- Easier To Do In Digital

Nonlinear Quantization



Non-Linear Quantization

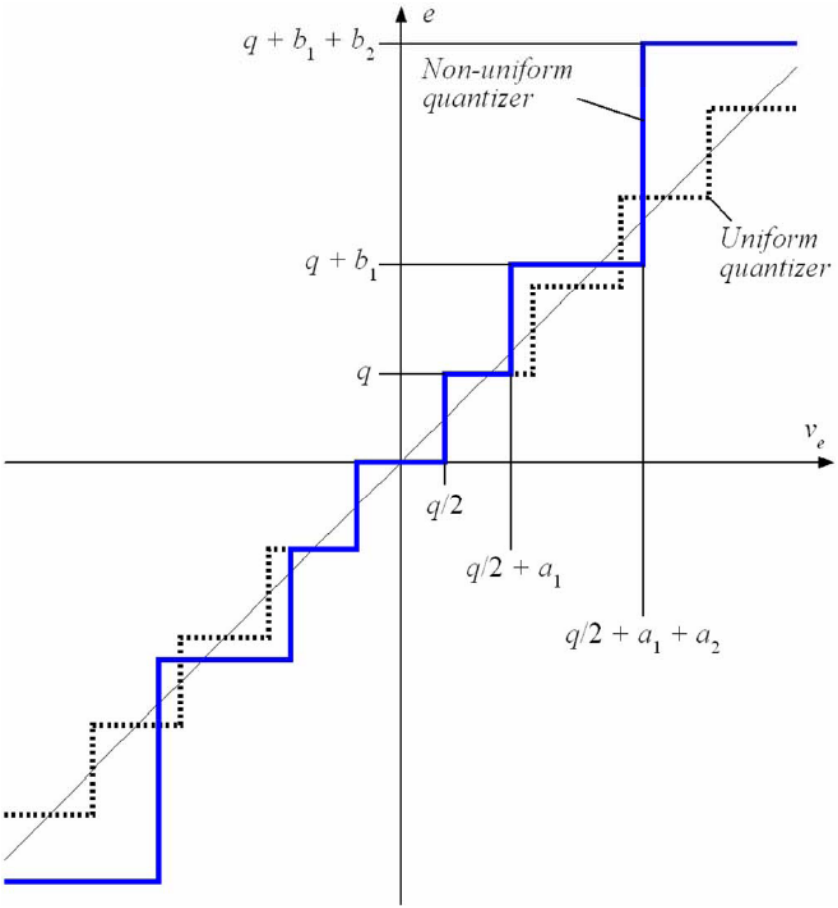


Fig. 2. A/D quantizer characteristics.

H. Hu*, et al.*, "Nonuniform A/D Quantization for Improved Dynamic Responses of Digitally Controlled DC–DC Converters," *Power Electronics, IEEE Transactions on,* vol. 23, pp. 1998-2005, 2008

Non-Linear Quantization

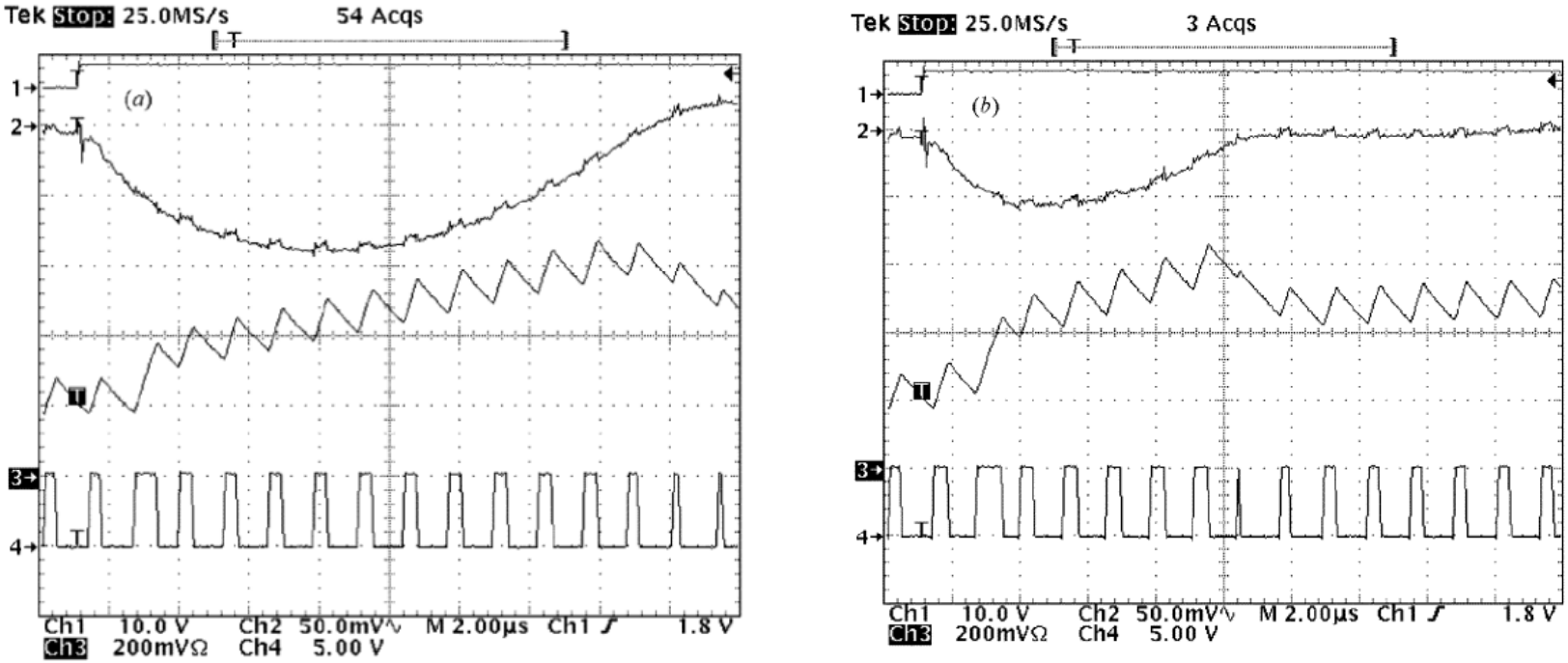
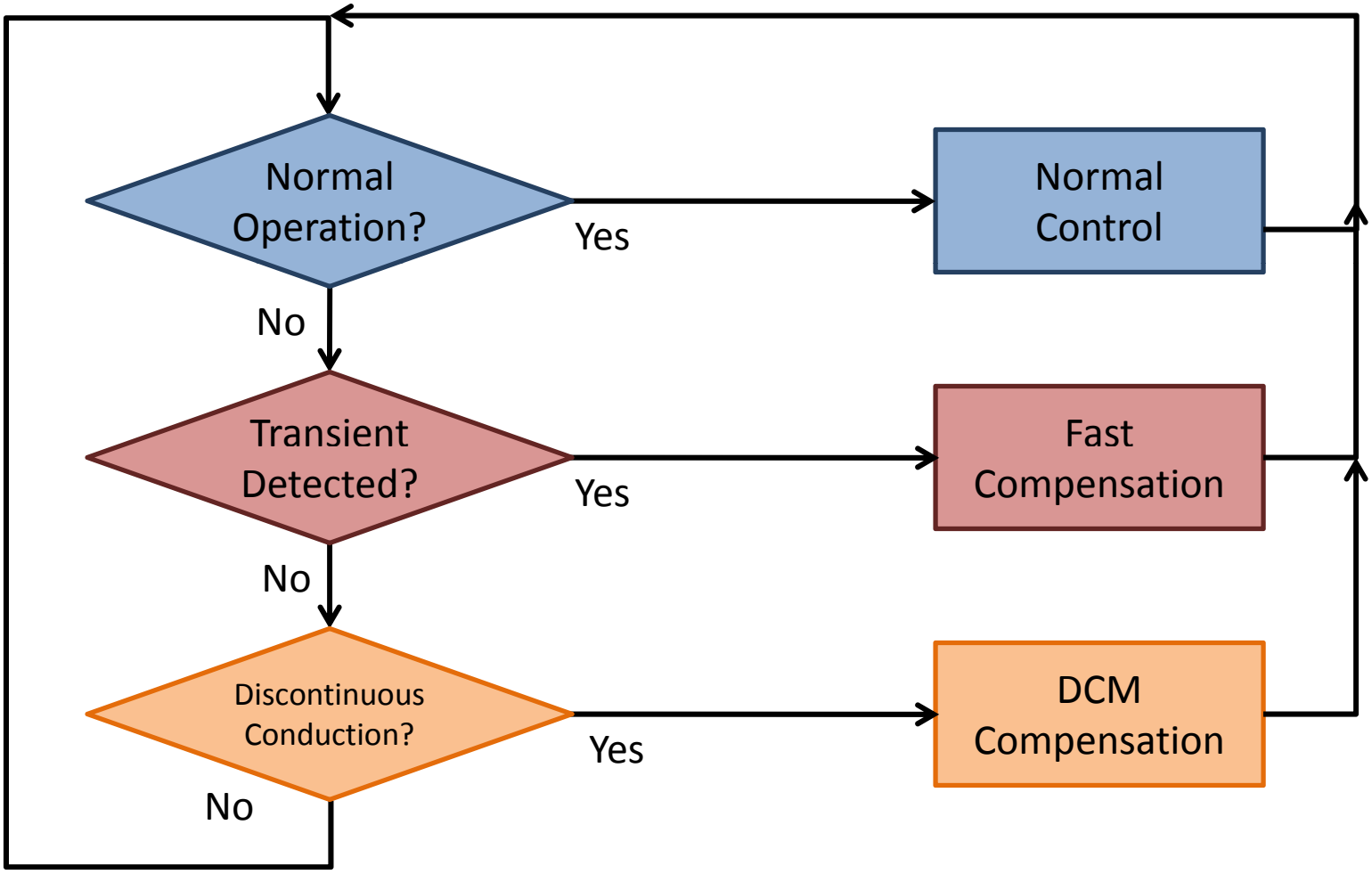


Fig. 15. Experimental step-load (50%-to-100%) transient responses: (a) uniform quantizer, (b) nonuniform design #1.
Top-to-bottom: load control signal, ac coupled output voltage v (t), inductor current i (t), switch-node voltage v (t).

H. Hu*, et al.*, "Nonuniform A/D Quantization for Improved Dynamic Responses of Digitally Controlled DC–DC Converters," *Power Electronics, IEEE Transactions on,* vol. 23, pp. 1998-2005, 2008

Variable Mode Controllers



Example

Non-Linear Transient Control Function

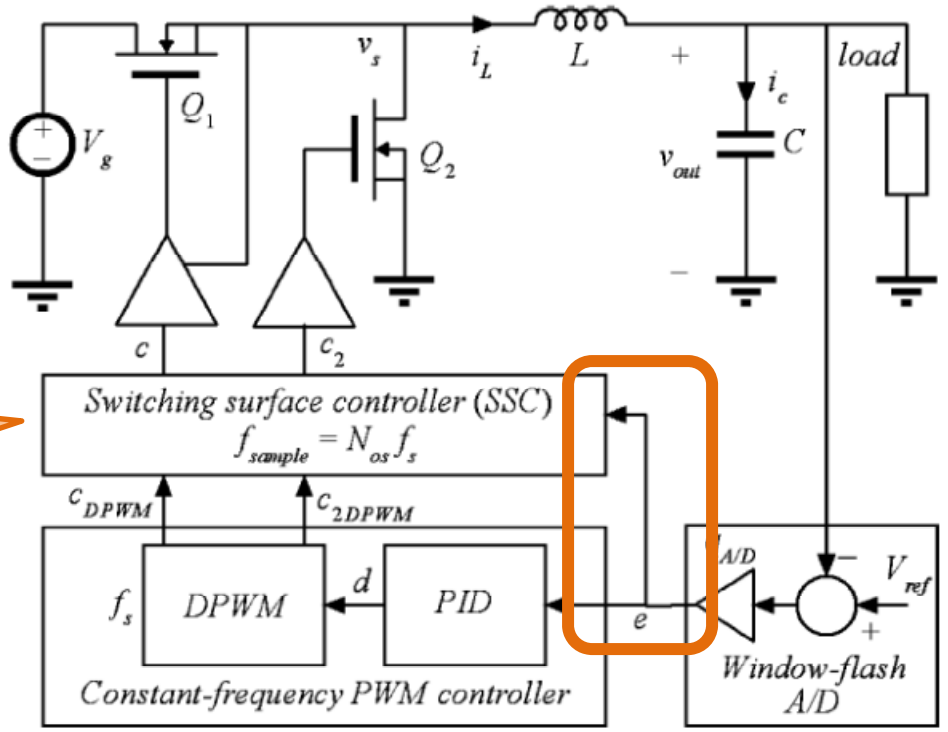


Fig. 4. Proximate time-optimal digital controller for a synchronous buck converter. Experimental prototype parameters: $V_g = 6.5$ V, $L = 1\ \mu$H, $C = 288\ \mu$F (ceramic), $R_{\text{esr}} = 1\text{m}\Omega$, $V_{\text{ref}} = 1.3$ V, $f_s = 780$ kHz, $I_{\text{load}} =$ 0-to-10 A, $q_{A/D} = 10$ mV, $N_{\text{os}} = 32, k = 32$.

V. Yousefzadeh, *et al.*, "Proximate Time-Optimal Digital Control for Synchronous Buck DC–DC Converters," *Power Electronics, IEEE Transactions on,* vol. 23, pp. 2018-2026, 2008

Example

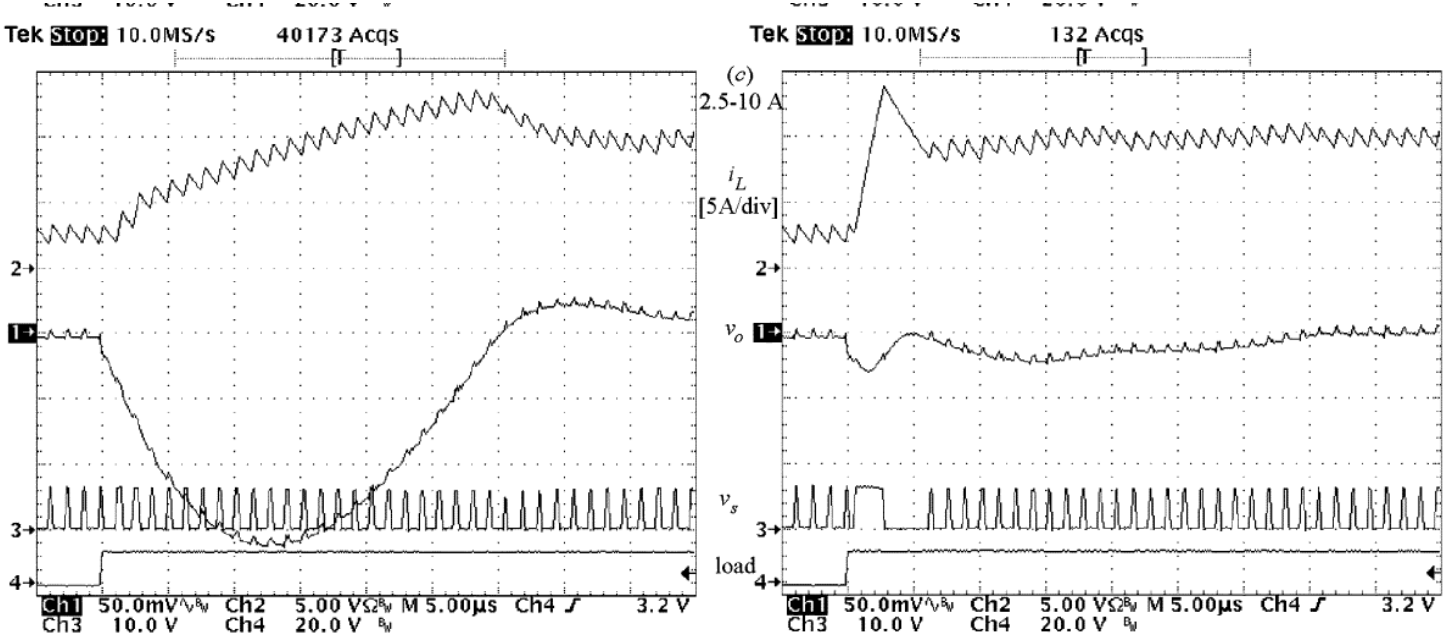


Fig. 7. Experimental step-load transient waveforms for the PWM controller with PID compensator (left) and the PTOD controller (right): (a) 7.5–10 A, (b) 5–10 A, (c) 2.5–10 A. The waveforms shown are, top-to-bottom: inductor current i_L (5 A/div), ac coupled output voltage v_{out} (50 mV/div), switch-node voltage v_s (10 V/div), and load control signal.

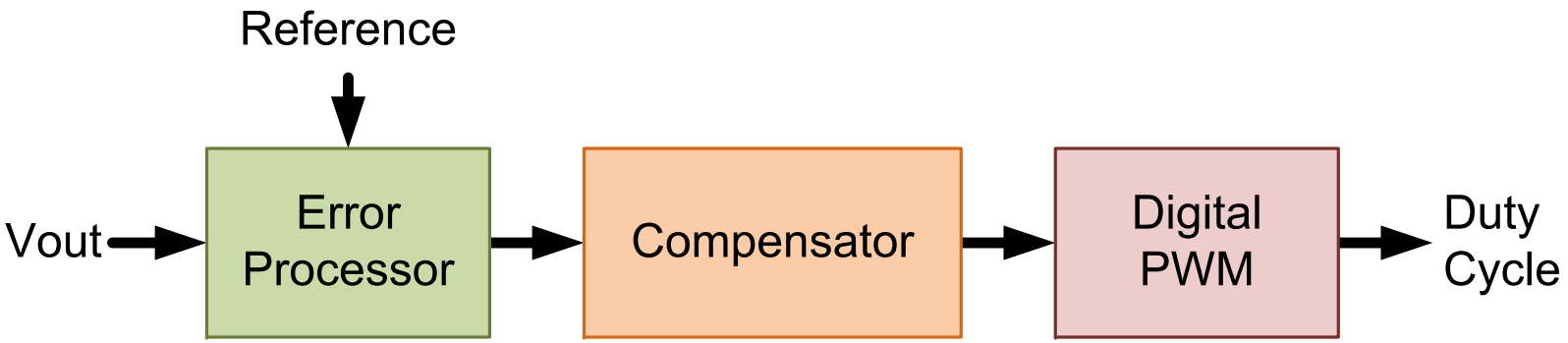
V. Yousefzadeh, *et al.*, "Proximate Time-Optimal Digital Control for Synchronous Buck DC–DC Converters," *Power Electronics, IEEE Transactions on,* vol. 23, pp. 2018-2026, 2008

Variable Mode Controllers

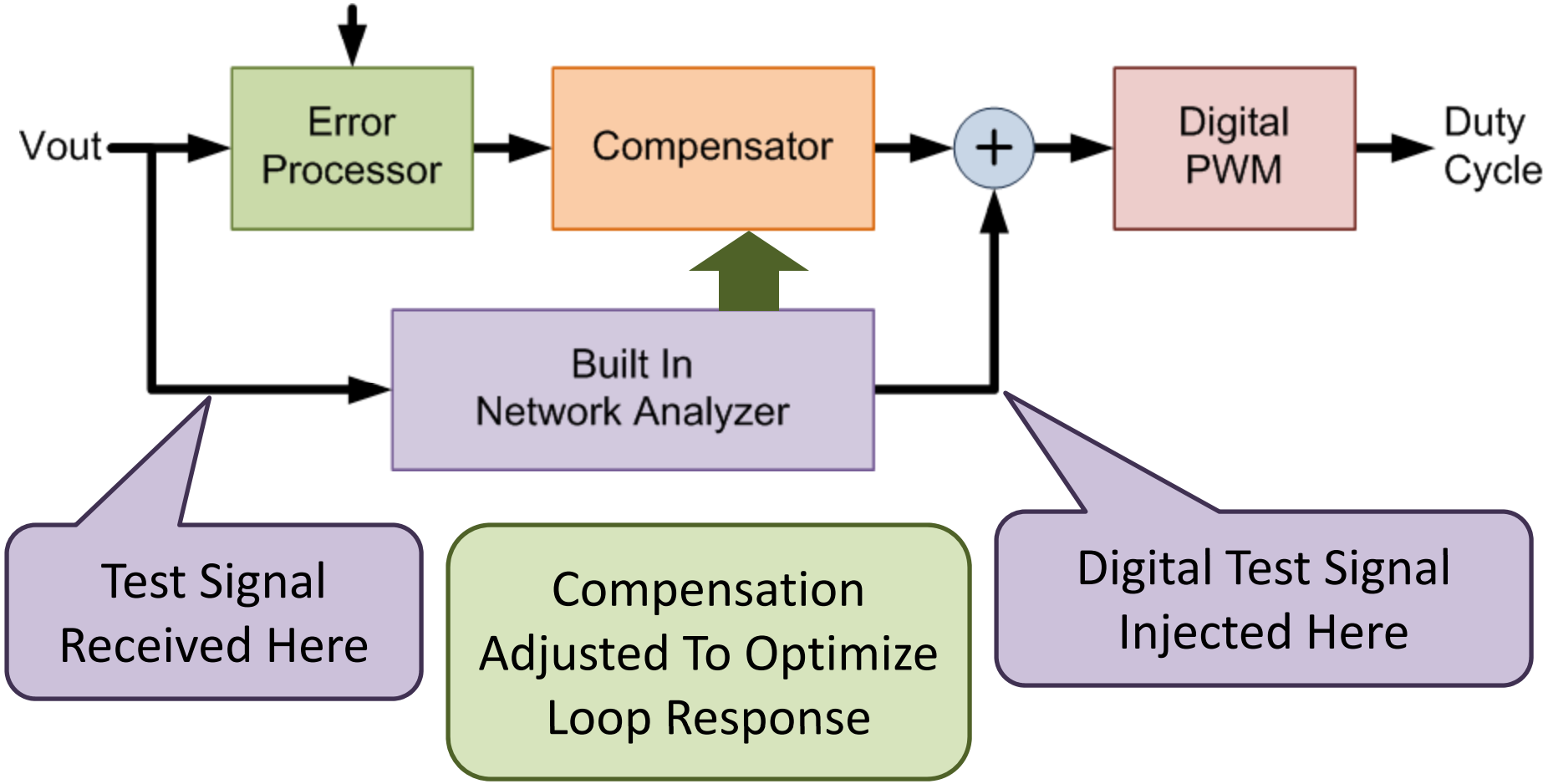
- How Is Transient Detected?
  - Detecting Small, Fast Variations Leads To Noise Sensitivity
  - Opposite Issue: Slow Response And Minimal Improvement In Response
- Shifting From One Mode To Another
  - Done Smoothly?
  - Lots Of Opportunity For Oscillations During Transitions

Self Tuning Control Loops

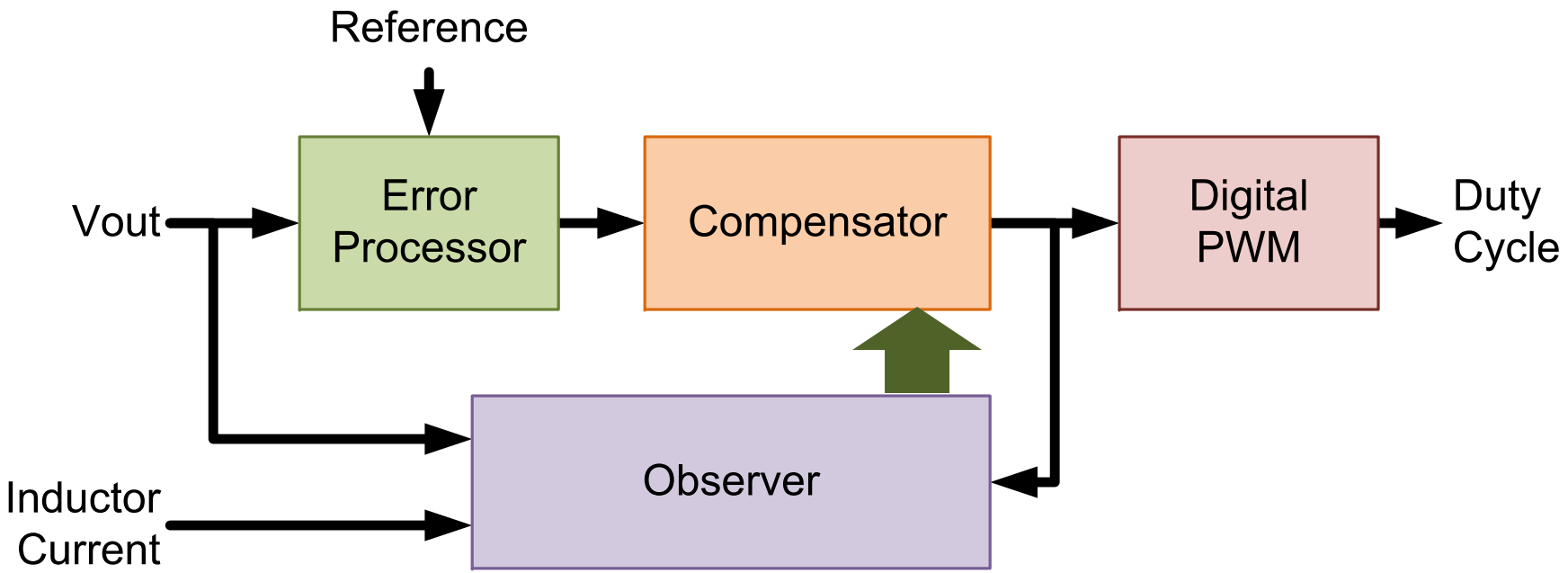
Start With The Basic Controller…



System Identification Method



Observer Method



Observer Uses Measurements To Determine The Power Stage Characteristics

Compensation Adjusted To Optimize Loop Response

EMBEDDED POWER LABS

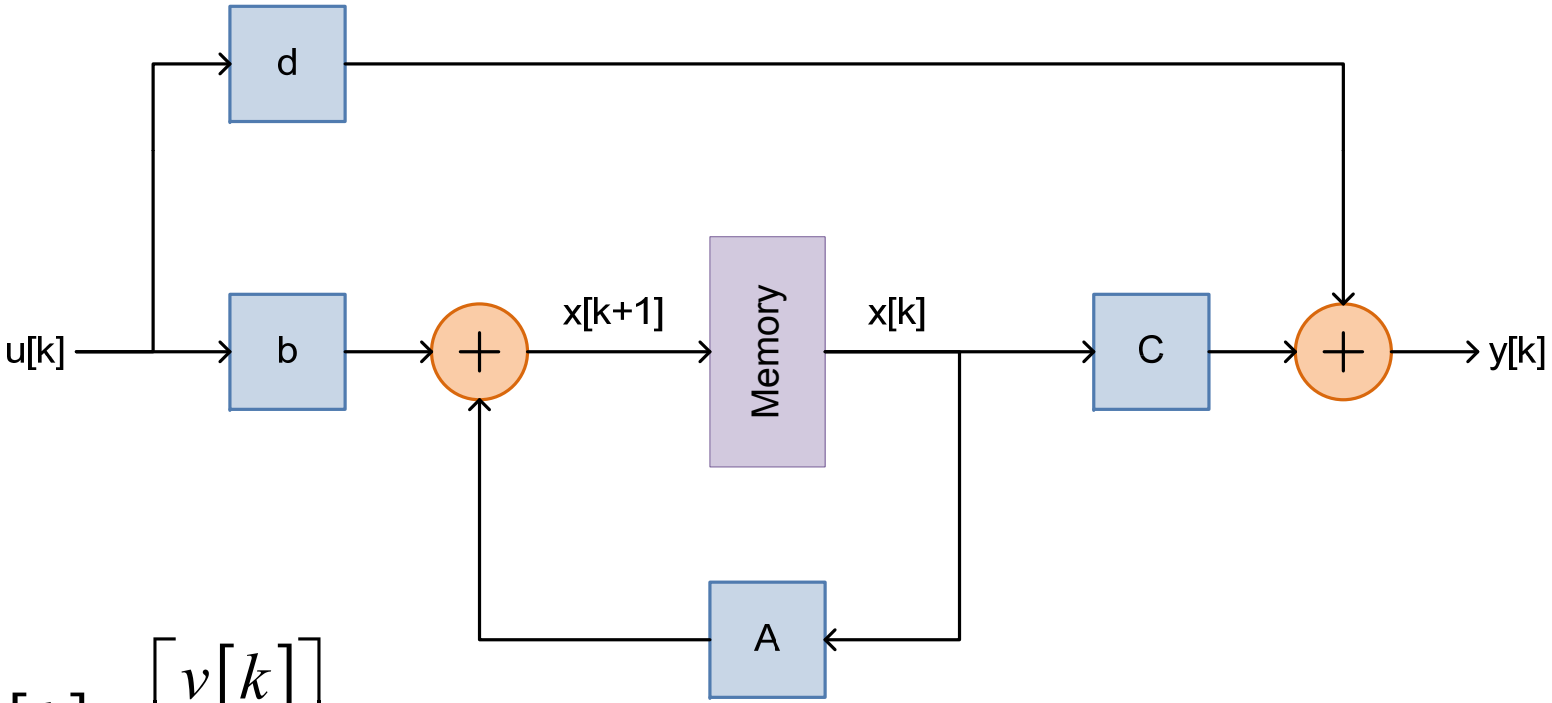
Musings On The Future

More Advanced Control

- Power Supply Control Has Not Been A Hot Topic Among Control Theorists
- What We Have Had:
  - Works Well
  - Is Inexpensive
- Cost Of Digital Has Discouraged R&D

Compensator: State Feedback

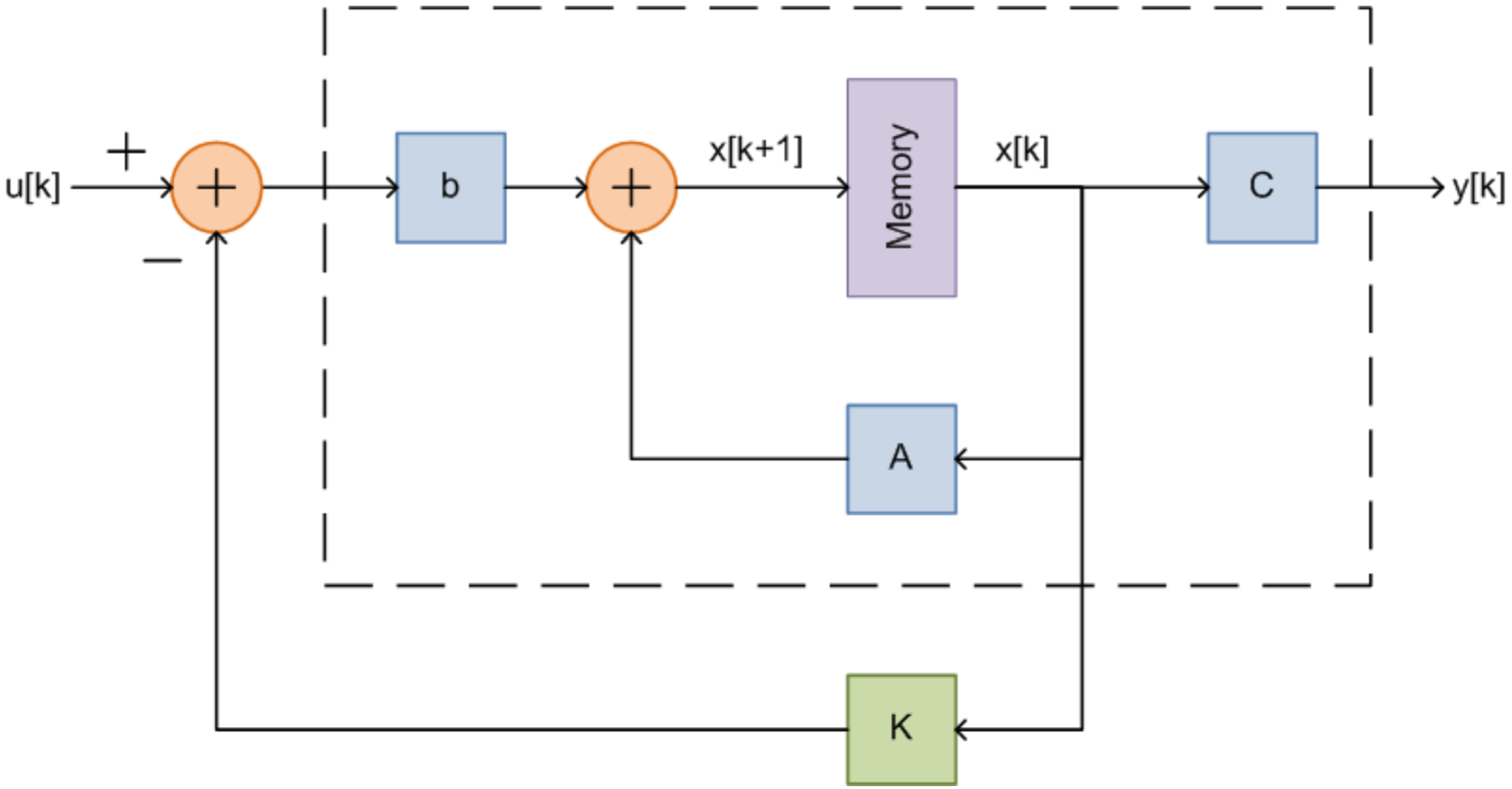
State Space Model Of A Discrete Time Plant



$$x[k] = \begin{bmatrix} v[k] \\ i[k] \end{bmatrix}$$

$$x_{k+1} = A \cdot x_k + b \cdot u_k \qquad y_k = C \cdot x_k + d \cdot u_k$$

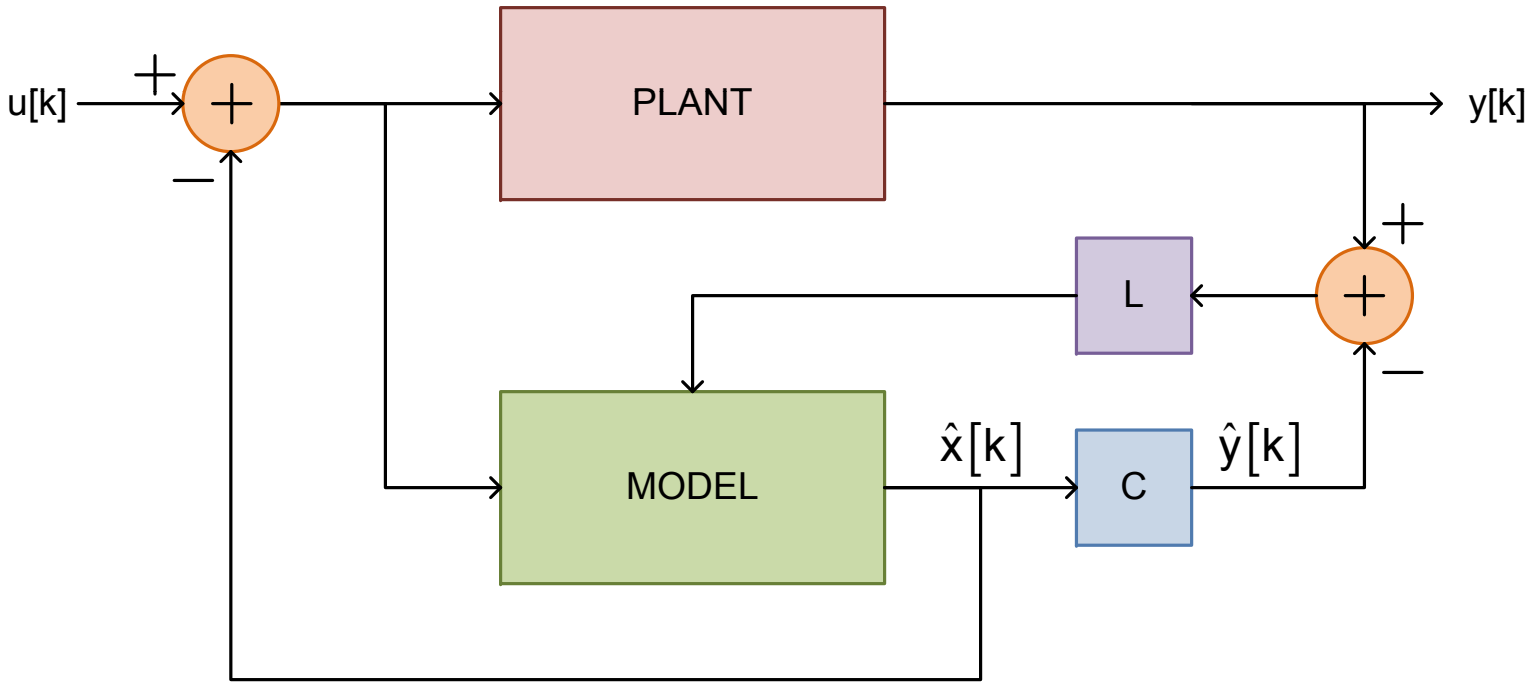
Compensator: State Feedback



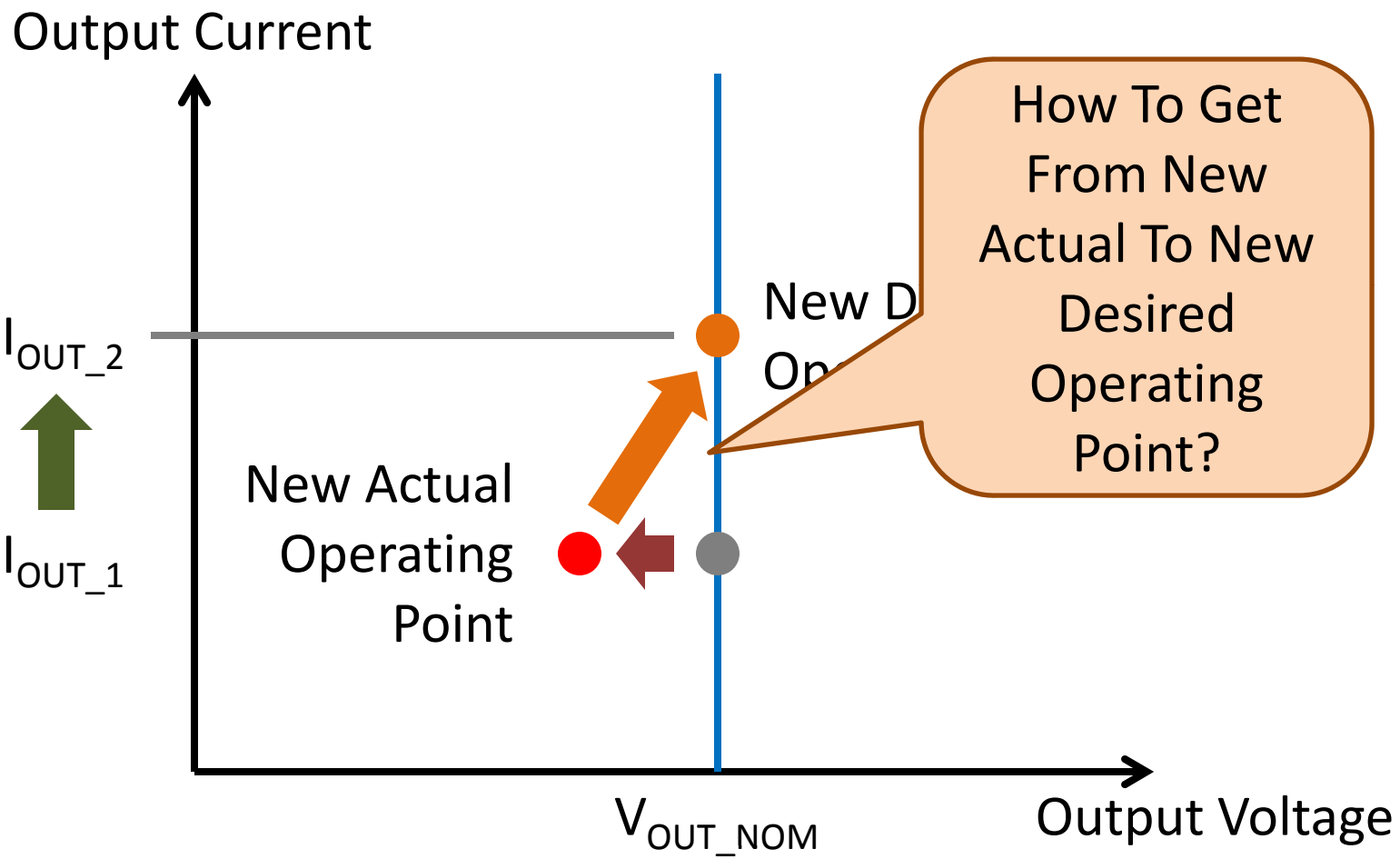
Works Same Way As Continuous Time State Feedback

Compensator: State Feedback

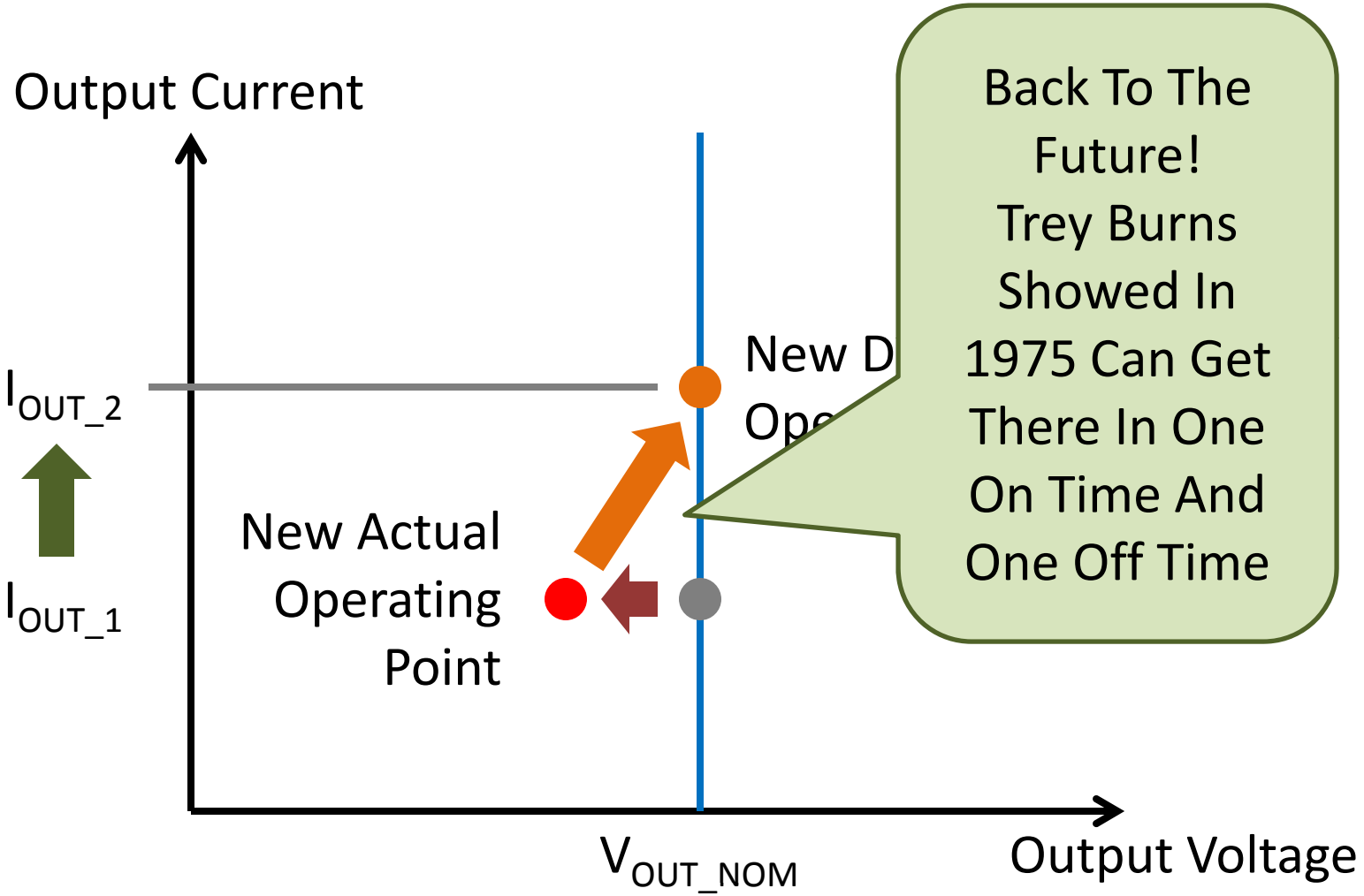
State Estimation Based Feedback



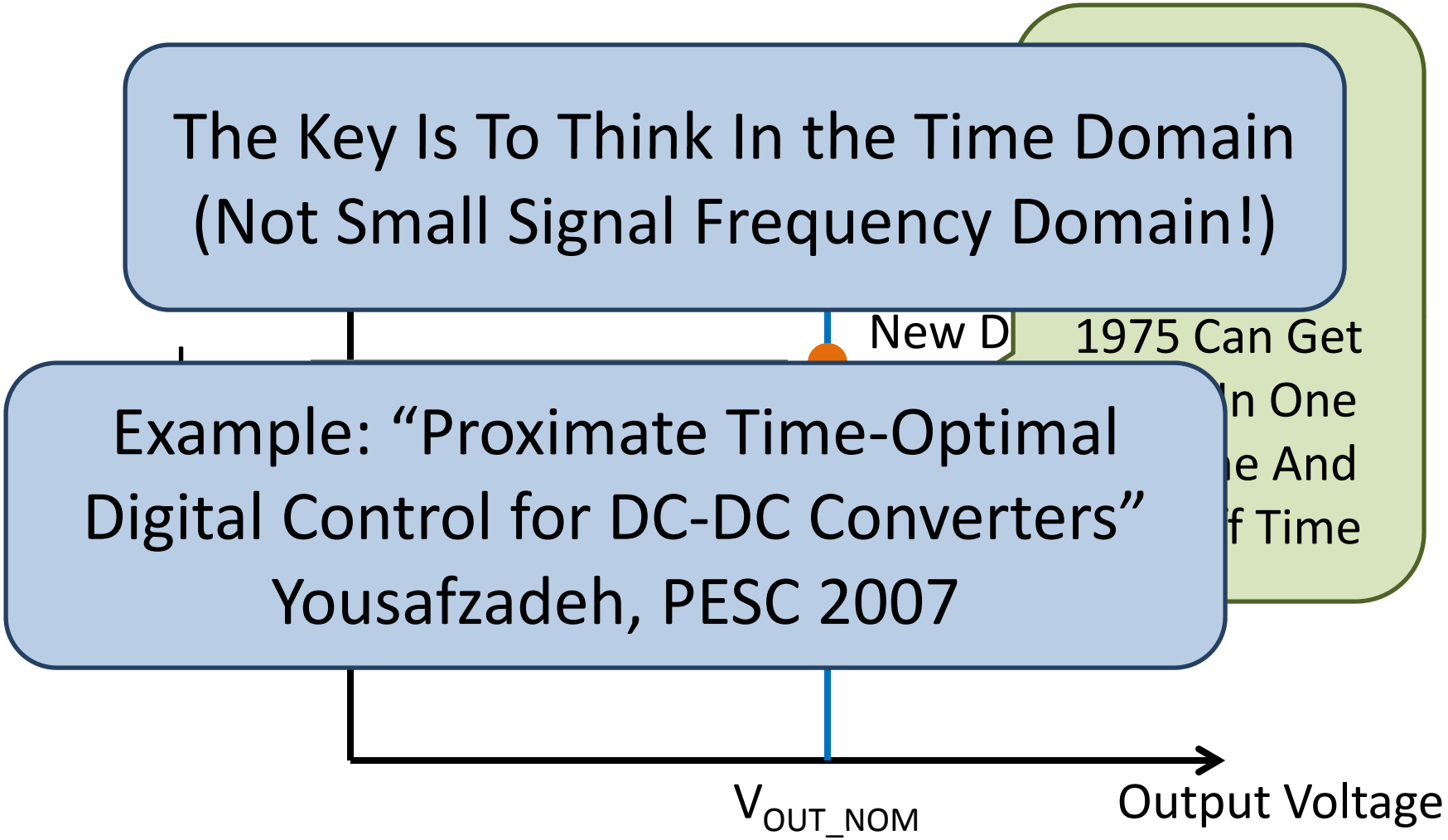
More Advanced Control



More Advanced Control



More Advanced Control



More Advanced Control

- We Are Ready To Start Applying More Advanced Control Techniques
  - State Feedback
  - Optimal Control
    - Linear Quadratic Regulators (LQR)
- Example: Cirasys
  - Uses Input-Output Linearization

Platforms

- ASICs Will Dominate Volume And Revenue
- Some Products Will Ship With DSCs
  - Low Volume Or Specialized Applications
- Hybrid Approaches Are TBD
  - Interesting But Waiting For The Killer Product
  - Cost Will Always Be An Issue
- FPGA Plus
  - Very Limited Production Volume

Barriers

- A/D Speed
- Sensor Speed & Accuracy For State Based Control
  - Including Current Mode Control
- DPWM Speed
- IP Minefield
- Increasing Switching Speed Driving Faster Controllers
- History
- Cost

Opportunities

- Decreasing Cost Of Silicon
- Control Engineers Bringing New Perspectives

Where Are We Going?

- Will We Find A Barren Desert Or
  A Rich, Fertile Wonderland?
- Could See Lots Of Experimental Products With No Commercial Success
- Could See Some Breakthrough Ideas
- I Predict A Very Interesting 3-5 Years As We Explore This New Frontier

About The Speaker

Bob White, Chief Engineer, Embedded Power Labs

- Bob has broad experience in designing power supplies, dc-dc converters and power systems for electronic equipment. He is widely recognized as an expert in power systems architecture and digital power. Bob is the principal author of the PMBus™ specifications. He is a well known speaker and author who has presented many papers and seminars at conferences such as the IEEE Applied Power Electronics Conference (IEEE APEC), the European Power Electronics Conference (EPE), the IEEE International Telecommunications Energy Conference (INTELEC) and the IEEE International Congress on Power Electronics (CIEP). In more than 30 years of professional experience, Bob has held managerial and individual contributor positions with Vu1 Corporation, Emerson Network Power/Artesyn Technologies/Zytec Corporation, AT&T Bell Labs/Power Systems, the Digital Equipment Corporation and General Electric.
- Bob has a BSEE from MIT (1980) and a MSEE from the Worcester Polytechnic Institute (1991).
- He is a Fellow of the IEEE, in which he has been active for more than 25 years. He was elected to the Power Electronics Society's executive committee three times, served two terms as the Society's Technical Vice President. Bob is well known for his key role in developing and supporting the IEEE Applied Power Electronics Conference (APEC). Bob was awarded the IEEE Third Millennium Medal in 2000 and the IEEE Power Electronic Society's Distinguished Service Award in 2002. The Power Sources Manufacturers Association (PSMA) recognized Bob in 2005 for his contributions to and leadership of the PSMA and APEC.