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EE115 Analog Circuits
Differential Pair

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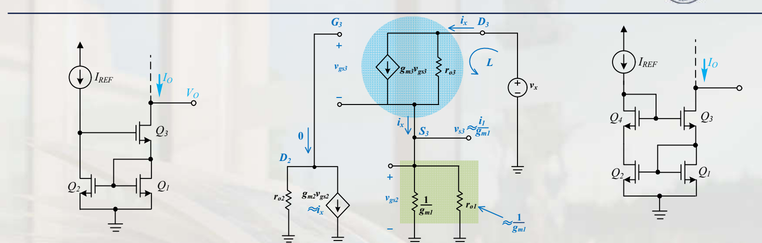
Outline

- Differential Amplifiers 1
  - MOS Differential Pair
- Reading: SEDTRA/SMITH book pages 576-594

Review: Wilson MOS Mirror



$R_o \equiv \frac{v_x}{i_x}$

$v_{gs2} \cong i_x/g_{m1}$

$g_{m2}v_{gs2} = g_{m2}\frac{i_x}{g_{m1}} = i_x$

$v_{g3} = v_{d2} = -i_x r_{o2}$

$v_{gs3} = v_{g3} - v_{s3} = v_{d2} - v_{gs2} = -i_x r_{o2} - \frac{i_x}{g_{m1}} \cong -i_x r_{o2}$

$g_{m3}v_{gs3} = -(g_{m3}r_{o2})i_x$

$v_x = (i_x + g_{m3}r_{o2}i_x)r_{o3} + \frac{i_x}{g_{m1}}$

$R_o \equiv \frac{v_x}{i_x} = (g_{m3}r_{o3})r_{o2} + r_{o3} + \frac{1}{g_{m1}} \cong (g_{m3}r_{o3})r_{o2}$

Why Differential?

- Differential circuits are much **less sensitive** to noises and interferences
- Differential configuration enables us to bias amplifiers and connect multiple stages **without using coupling or bypass capacitors**
- Differential amplifiers are widely used in ICs
  - Excellent **matching** of transistors, which is critical for differential circuits
  - Differential circuits require **more transistors**-not an issue for IC

MOS Differential-Pair

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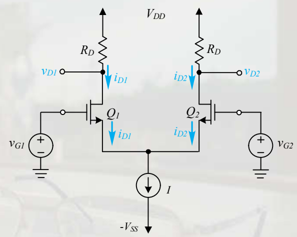
- Two **matched** MOS transistors Common current bias.
- **Differential signals** applied to v_{G1} and v_{G2} (**equal amplitude but opposite sign**).
- Differential outputs are produced at v_{D1} and v_{D2}.
- Note: in differential configuration with 0 differential input, V_{GS} is fixed for both Q_1 and Q_2.

Basic Config.



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Continued (Common-Mode Input)

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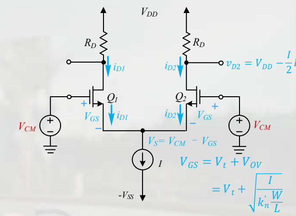
- The common voltages applied to both Q_1 and Q_2 are referred to as V_{CM}.
- Common mode inputs usually come from **noises or interferences**.
- Differential pair should **reject** V_{CM}:
  - Since $V_{GS1} = V_{GS2} = V_t + \sqrt{I/k_n}$ is fixed
  - V_{CM} simply changes the voltage at Source, V_S
- The drain currents remain fixed:

$$v_{D1} = v_{D2} = V_{DD} - \frac{I}{2}R_D$$

Differential Pair Rejects Common-Mode Inputs



$$V_{CMmin} = -V_{SS} + V_{CS} + V_{GS} = -V_{SS} + V_{CS} + V_t + V_{OV}$$

$$V_{CMmax} = V_D + V_t = V_{DD} - \frac{I}{2}R_D + V_t$$

V_{CS}: Minimum voltage across current source

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Example: diff pair

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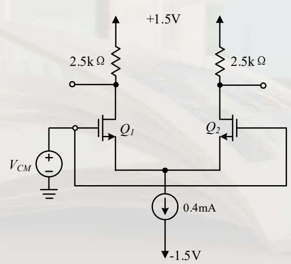
$V_{DD} = V_{SS} = 1.5\text{V}$, $I = 0.4\text{mA}$, $R_D = 2.5\text{k}\Omega$. Minimum voltage across current source $V_{CS} = 0.4\text{V}$. For Q_1 and Q_2: $k_n = 4\ \text{mA/V}^2$, $V_{tn} = 0.5\text{V}$.

(a) Find V_{OV} and V_{GS} for each transistor



Solution:

$$I_{D1} = I_{D2} = \frac{I}{2}$$

$$\frac{I}{2} = \frac{1}{2}k_n'(W/L)V_{OV}^2$$

$$\frac{0.4}{2} = \frac{1}{2} \times 4V_{OV}^2$$

$$V_{OV} = 0.316V$$

$$V_{GS} = V_t + V_{OV} = 0.5 + 0.316 \simeq 0.82V$$

Continued

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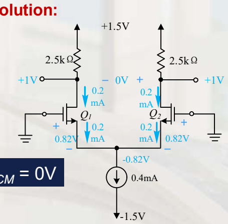
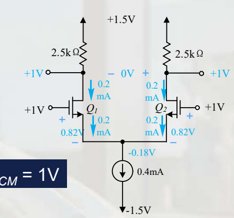
(b)Find V_S, I_{D1}, I_{D2}, V_{D1}, V_{D2} for 3 different V_{CM} below:

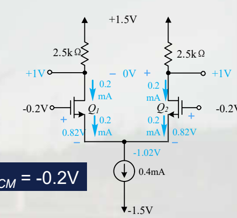
Solution:



$V_{CM} = 0\text{V}$



$V_{CM} = 1\text{V}$



$V_{CM} = -0.2\text{V}$

$$V_S = V_G - V_{GS} = 0 - 0.82 = -0.82V$$

$$I_{D1} = I_{D2} = \frac{I}{2} = 0.2mA$$

$$V_{D1} = V_{D2} = V_{DD} - \frac{I}{2}R_D = 1.5 - 0.2 \times 2.5 = +1V$$

$$V_O = V_{D2} - V_{D1} = 0V$$

$$V_S = V_G - V_{GS} = 1 - 0.82 = +0.18V$$

$$I_{D1} = I_{D2} = \frac{I}{2} = 0.2mA$$

$$V_{D1} = V_{D2} = V_{DD} - \frac{I}{2}R_D = 1.5 - 0.2 \times 2.5 = +1V$$

$$V_O = V_{D2} - V_{D1} = 0V$$

$$V_S = V_G - V_{GS} = -0.2 - 0.82 = -1.02V$$

$$V_{CS} = -V_S - (-V_{SS}) = -1.02 + 1.5 = +0.48V$$

$$I_{D1} = I_{D2} = \frac{I}{2} = 0.2mA$$

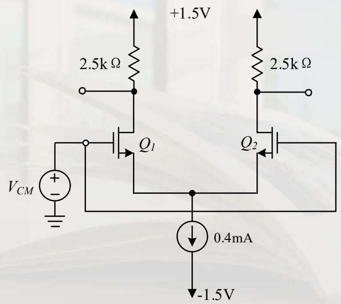
$$V_{D1} = V_{D2} = V_{DD} - \frac{I}{2}R_D = +1V$$

$$V_O = V_{D2} - V_{D1} = 0V$$

I_{D1}, I_{D2}, V_{D1}, V_{D2}, V_o are unchanged

Continued

(c) What is the highest/lowest permitted values of V_{CM}?



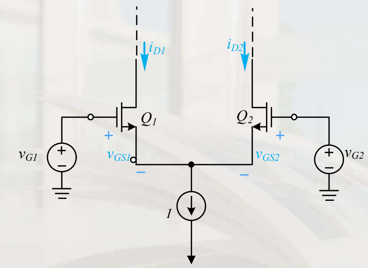
Solution:

$$V_{CMmax} = V_t + V_D$$
$$= 0.5 + 1 = +1.5V$$

$$V_{CMmin} = -V_{SS} + V_{CS} + V_{GS}$$
$$= -1.5 + 0.4 + 0.82 = -0.28V$$

$$-0.28V \le V_{CM} \le +1.5V$$

Operation with Differential Input Voltage



$$i_{D1} = \frac{1}{2}k_n'\frac{W}{L}(v_{GS1} - V_t)^2$$

$$i_{D2} = \frac{1}{2}k_n'\frac{W}{L}(v_{GS2} - V_t)^2$$

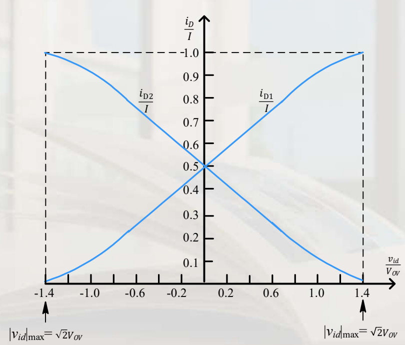
$$\sqrt{i_{D1}} - \sqrt{i_{D2}} = \sqrt{\frac{1}{2}k_n'\frac{W}{L}} \cdot v_{id}$$

$$i_{D1} + i_{D2} = I$$

$$i_{D1} = \frac{I}{2} + \sqrt{k_n'\frac{W}{L}I} \cdot \left(\frac{v_{id}}{2}\right) \cdot \sqrt{1 - \frac{(v_{id}/2)^2}{I/k_n'\frac{W}{L}}}$$

$$i_{D2} = \frac{I}{2} - \sqrt{k_n'\frac{W}{L}I} \cdot \left(\frac{v_{id}}{2}\right) \cdot \sqrt{1 - \frac{(v_{id}/2)^2}{I/k_n'\frac{W}{L}}}$$

Operation with Differential Input Voltage



$$i_{D1} = \frac{I}{2} + \sqrt{k_n'\frac{W}{L}I} \cdot \left(\frac{v_{id}}{2}\right) \cdot \sqrt{1 - \frac{(v_{id}/2)^2}{I/k_n'\frac{W}{L}}}$$

$$i_{D2} = \frac{I}{2} - \sqrt{k_n'\frac{W}{L}I} \cdot \left(\frac{v_{id}}{2}\right) \cdot \sqrt{1 - \frac{(v_{id}/2)^2}{I/k_n'\frac{W}{L}}}$$

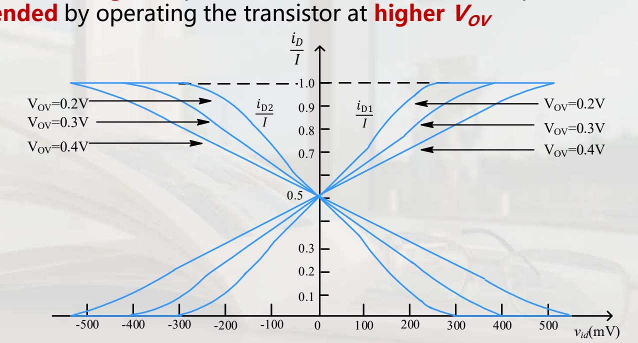
For small v_{id}, it could be linear:

$$i_{D1} \cong \frac{I}{2} + \left(\frac{I}{V_{OV}}\right)\left(\frac{v_{id}}{2}\right)$$

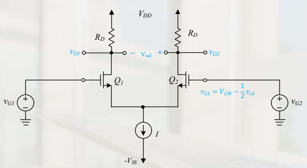
$$i_{D2} \cong \frac{I}{2} - \left(\frac{I}{V_{OV}}\right)\left(\frac{v_{id}}{2}\right)$$

Current of Differential Pair for Various V_{OV}

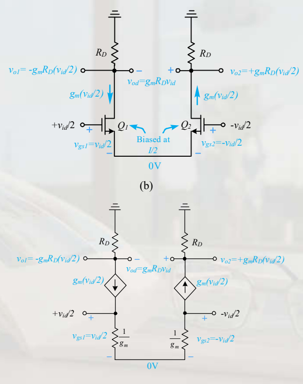
- The **linear range** of operation of the MOS differential pair can be **extended** by operating the transistor at **higher V_{OV}**



Small Signal Operation



(a)



(b)

(c)

- For **differential AC small signal**, the differential pair is **anti-symmetric**. The potential at the mid point is zero. This is called **Virtual Ground**
- This virtual ground is obtained **without bypass capacitor** -> much smaller area and better frequency response

$$v_{G1} = V_{CM} + \frac{1}{2}v_{id}$$

$$v_{G2} = V_{CM} - \frac{1}{2}v_{id}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$A_d \equiv \frac{v_{od}}{v_{id}} = \frac{v_{o2} - v_{o1}}{v_{id}} = g_m R_D$$

Differential Half Circuit

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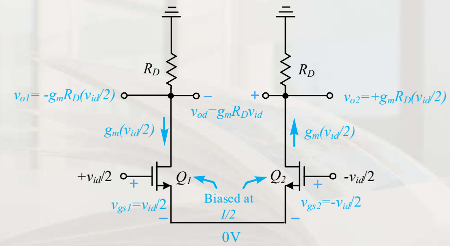
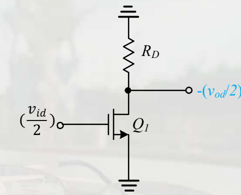
- Because the two halves of the circuits are **anti-symmetric**, and **source** is at **virtual ground**, we can simplify and just analyze the **half circuit**





Equivalent half circuit
If consider r_o:

$$A_d = g_m(R_D || r_o)$$

Differential Amplifier w/ Current-Source Loads

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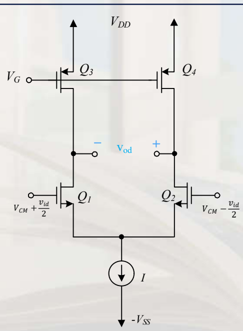
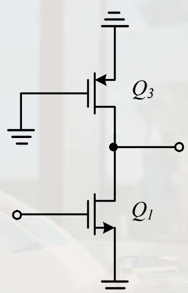


(a)



(b)

$$A_d \equiv \frac{v_{od}}{v_{id}} = g_{m1}(r_{o1} || r_{o3})$$

Cascode Differential Amplifier

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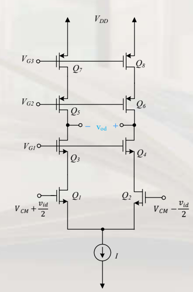
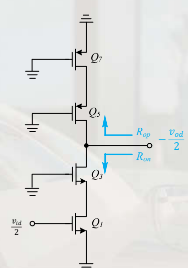
- Cascode configurations for both amplifying transistors and current source loads.


(a)


(b)

$$A_d \equiv \frac{v_{od}}{v_{id}} = g_{m1}(R_{on} || R_{op})$$

$$R_{on} = (g_{m3} r_{o3}) r_{o1}$$

$$R_{op} \equiv (g_{m5} r_{o5}) r_{o7}$$