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EE115 Analog Circuits Frequency Response

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电力电子与可再生能源系统

Outline

■ High Frequency Response 2

- Frequency Response of Common-Source Amplifiers
- Open-Circuit Time Constants (OCTC) Method
- High Frequency Response of Common Gate Amplifiers

■ Reading: SEDTRA/SMITH book pages 686-715

Review: Discrete vs Integrated Circuits

Discrete Circuits

- Resistors and capacitors are frequently used
- AC coupled with **coupling capacitors**
- High DC power supply voltage
- Transistor choice limited to available parts
- Mostly BJT, some MOS

IC

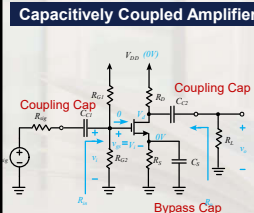
- Use mostly transistors
 - Resistors and capacitors occupy **too much areas**
- Mostly **DC coupled** (without capacitors)
- Low DC power supply voltage ($\sim 1V$)
- Can vary device size
- Predominantly CMOS
 - BiCMOS provides BJT

Review: CS Amplifier with Bias Circuit

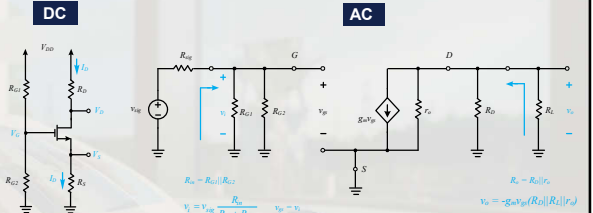
- Both coupling and bypass capacitors are DC-open and AC-short

Capacitively Coupled Amplifier

DC



AC

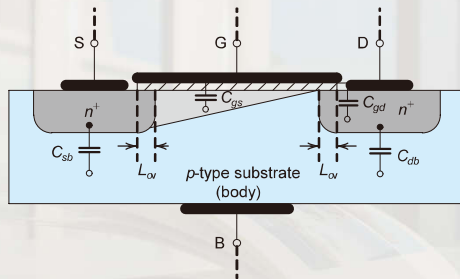


$$R_{in} = R_{G1} || R_{G2}$$

$$G_v = -\frac{R_{in}}{R_{in} + R_{sig}} g_m (R_D || R_L || r_o)$$

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Review: Cross Section of MOSFET w/ Internal Cap.



■ MOSFET has several internal capacitances, which take time to charge/discharge, limiting the **transistor speed**.

■ Gate/Source capacitance:

$$C_{gs} = \frac{2}{3} WLC_{ox} + C_{ov}$$

where

- $C_{ox} = \epsilon_{ox}/t_{ox}$ [F/cm^2]
- W: Transistor width
- L: Gate Length
- $C_{ov} = WL_{ov}C_{ox}$
- L_{ov} : overlap between Gate/Source or Gate/Drain, typically 0.05-0.1L

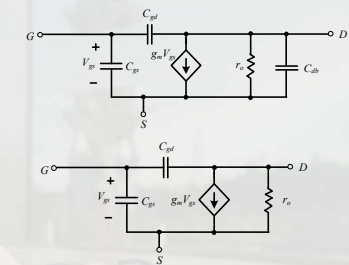
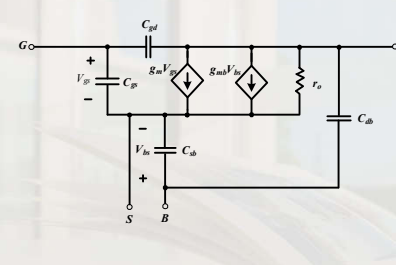
■ Gate/Drain capacitance:

$$C_{gd} = C_{ov}$$

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Review: MOSFET High-f Equivalent-Circuit Model



■ Source is connected to the body (no **body effect**)

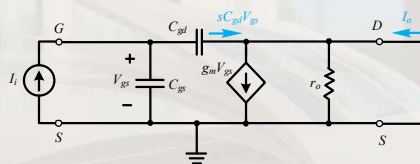
■ Capacitance between drain and body C_{db} neglected

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Review: Unity-Gain Frequency, f_T 

■ f_T : frequency at which short-circuit current gain =1

- FoM for transistor speed
- Drain is grounded (short-circuit load)



$$I_o = g_m V_{gs} - sC_{gd} V_{gs} = (g_m - sC_{gd}) V_{gs}$$

$$V_{gs} = \frac{I_i}{s(C_{gs} + C_{gd})}$$

$$\text{Combine: } \frac{I_o}{I_i} = \frac{g_m - sC_{gd}}{s(C_{gs} + C_{gd})}$$

$$\left| \frac{I_o}{I_i} \right| (s = j\omega_T) \cong \frac{g_m}{\omega_T (C_{gs} + C_{gd})} = 1$$

$$\text{Unity gain freq.: } \omega_T = \frac{g_m}{(C_{gs} + C_{gd})}$$

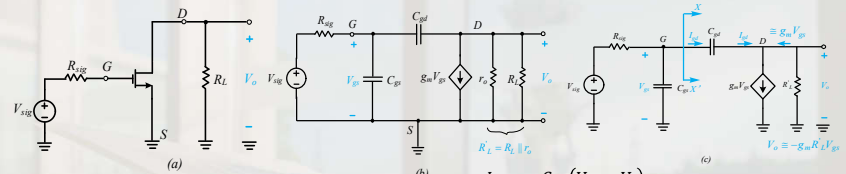
$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

As gate length reduces in advanced technology, C_{gs} reduces and f_T increases

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High Frequency Response of CS Amplifier



■ Middle band gain, A_M

➢ C_{gd} , C_{gs} ignored

$$R'_L = R_L || r_o \quad A_M = \frac{V_o}{V_{sig}} = -g_m R'_L$$

$$\begin{aligned} I_{gd} &= sC_{gd}(V_{gs} - V_o) \\ &= sC_{gd}[V_{gs} - (-g_m R'_L V_{gs})] \\ &= sC_{gd}(1 + g_m R'_L)V_{gs} \end{aligned}$$

$$V_o = -(g_m V_{gs} - I_{gd})R'_L$$

Middle band

$$\frac{V_o}{V_{gs}} = -[g_m - sC_{gd}(1 + g_m R'_L)]R'_L \cong -g_m R'_L$$

Tedious, not intuitive.

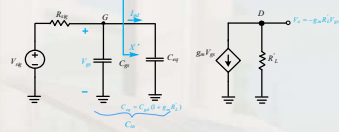
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Miller Capacitance



Equivalent Circuit



$$I_{gd} = sC_{gd}(1 + g_m R'_L)V_{gs} = sC_{eq}V_{gs}$$

$$C_{eq} = C_{gd}(1 + g_m R'_L)$$

Miller effect, $(1 + g_m R'_L)$ is known as Miller multiplier

$$V_{gs} = \frac{1}{1 + \frac{s}{\omega_p}} V_{sig}$$

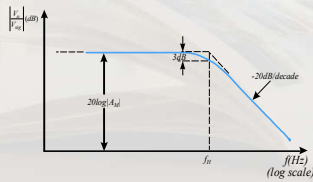
Where, ω_p is the pole frequency of the STC circuit

$$\omega_p = 1/(C_{in}R_{sig}), C_{in} = C_{gs} + C_{eq} = C_{gs} + C_{gd}(1 + g_m R'_L)$$

$$\frac{V_o}{V_{sig}} = -\frac{g_m R'_L}{1 + \frac{s}{\omega_p}} = \frac{A_M}{1 + \frac{s}{\omega_H}}$$

$$\omega_H = \omega_p = 1/(C_{in}R_{sig})$$

■ Large input capacitance in CS amplifier due to Miller effect, which greatly reduced the bandwidth of CS amp

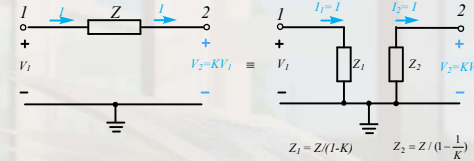


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Miller's Theorem



Miller Equivalent



$$\text{If } V_2 = KV_1$$

$$I = \frac{V_1 - KV_1}{Z}$$

Input side:

$$I_1 = \frac{V_1}{Z_1} = I = \frac{V_1 - KV_1}{Z}$$

$$Z_1 = \frac{Z}{(1-K)}$$

Output side:

$$I_2 = \frac{0 - V_2}{Z_2} = \frac{0 - KV_1}{Z_2} = I = \frac{V_1 - KV_1}{Z}$$

$$Z_2 = \frac{Z}{(1 - \frac{1}{K})}$$



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Open-Circuit Time Constant (OCTC) Method for High Cut-off Frequency

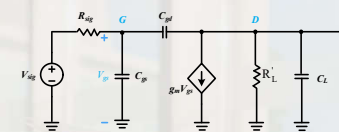


- Replace all **capacitors** by **open circuit**
- **Signal source** becomes **zero**
- Consider one capacitor at a time, find resistance R_i seen by the i-th capacitor, C_i
- $\omega_H \approx \frac{1}{\sum_i C_i R_i}$



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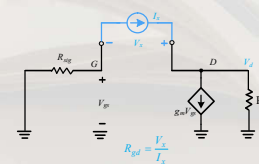
Applying OCTC to CS Amplifier



Capacitor = Open

Signal source = 0

2. Only consider C_{gd} , find R_{gd}



1. Only consider C_{gs} , find R_{gs}



$$V_{gs} = -I_x R_{sig}$$

$$V_d = V_x + V_{gs}$$

$$I_x = g_m V_{gs} + \frac{V_d}{R'_L}$$

$$I_x = g_m V_{gs} + \frac{V_x + V_{gs}}{R'_L}$$

$$R_{gd} \equiv \frac{V_x}{I_x} = R_{sig}(1 + g_m R'_L) + R'_L \dots (2)$$

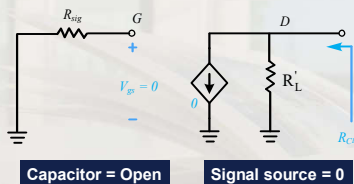
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Applying OCTC to CS Amplifier



$$R_{gs} = R_{sig} \cdots (1)$$

$$R_{gd} = R_{sig}(1 + g_m R_L') + R_L' \cdots (2)$$



3. Only consider C_L , find R_{CL}

$$R_{CL} = R_L' \cdots (3)$$

$$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{CL}$$

$$= C_{gs}R_{sig} + C_{gd}[R_{sig}(1 + g_m R_L') + R_L'] + C_L R_L'$$

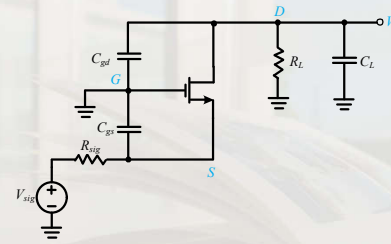
$$f_H = \frac{1}{2\pi\tau_H}$$

$$\tau_H = [C_{gs} + C_{gd}(1 + g_m R_L')]R_{sig} + (C_{gd} + C_L)R_L'$$

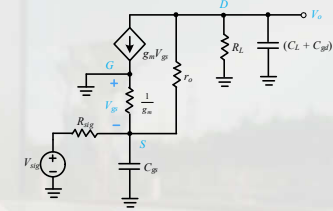
- Time constant from **input** port of Miller Equivalent Circuit
- Time constant from **output** port of Miller Equivalent Circuit

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High f Response of Common Gate Amplifier



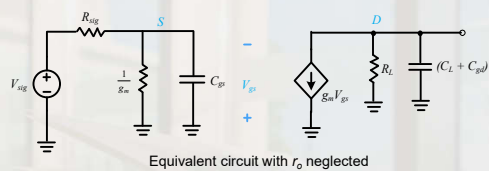
(a) CG amplifier with cap.(ac equivalent)



(b) Equivalent ac small signal model

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High f Response of Discrete Circuit CG Amplifier



- Discrete circuit CG: early effect can be ignored
- No Miller effect** since both capacitance are grounded
- The dominant term is likely to be $C_{gs}(R_{sig} || \frac{1}{g_m})$, which is small $\rightarrow$ High f_H
 - Common-Gate is a **broadband** amplifier

Why early effect can be ignored in discrete circuit?

$$f_{p1} = \frac{1}{2\pi C_{gs} (R_{sig} || \frac{1}{g_m})}$$

$$\tau_{gs} = C_{gs} (R_{sig} || \frac{1}{g_m}) = \frac{1}{2\pi f_{p1}}$$

$$f_{p2} = \frac{1}{2\pi (C_{gd} + C_L) R_L}$$

$$\tau_{gd} = (C_L + C_{gd}) R_L = \frac{1}{2\pi f_{p2}}$$

$$\tau_H = C_{gs} (R_{sig} || \frac{1}{g_m}) + (C_L + C_{gd}) R_L$$

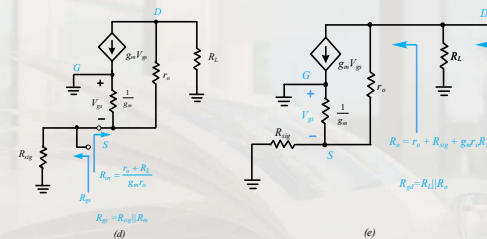
$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{\frac{1}{f_{p1}} + \frac{1}{f_{p2}}}$$

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High f Response of IC CG Amplifier



- IC CG: early effect must be **considered**.
- Apply **OCTC** method.



Capacitor = Open

Signal source = 0

$$R_{gs} = R_{sig} || R_{in}$$

$$R_{in} = \frac{r_o + R_L}{1 + g_m r_o} \approx \frac{r_o + R_L}{g_m r_o}$$

$$R_{gd} = R_L || R_o$$

$$R_o = r_o + R_{sig} + g_m r_o R_{sig}$$

$$\tau_H = \tau_{gs} + \tau_{gd}$$

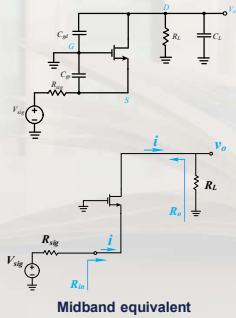
$$f_H = \frac{1}{2\pi\tau_H}$$

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Example: frequency response of CG amplifier



Consider a CG amplifier with $g_m = 2\text{mA/V}$, $r_o = 20\text{k}\Omega$, $C_{gs} = 20\text{fF}$, $C_{gd} = 5\text{fF}$, $C_L = 25\text{fF}$, $R_{sig} = 20\text{k}\Omega$, and $R_L = 20\text{k}\Omega$. Determine the input resistance, the midband gain, and the upper 3-dB frequency f_H .



Solution:

$$v_o = iR_L$$

$$v_{sig} = i(R_{sig} + R_{in})$$

$$G_v = \frac{v_o}{v_{sig}} = \frac{R_L}{R_{sig} + R_{in}}$$

$$R_{in} = \frac{r_o + R_L}{1 + g_m r_o} = \frac{20 + 20}{1 + (2 \times 20)} = 0.98\text{k}\Omega$$

$$G_v = \frac{20}{20 + 0.98} = \frac{0.95\text{V}}{\text{V}}$$

$$R_{gs} = R_{sig} \parallel R_{in} = 20 \parallel 0.98 = 0.93\text{k}\Omega$$

$$R_{gd} = R_L \parallel R_o$$

$$R_o = r_o + R_{sig} + (g_m r_o) R_{sig} = 20 + 20 + 40 \times 20 = 840\text{k}\Omega$$

$$R_{gd} = 20 \parallel 840 = 19.5\text{k}\Omega$$

$$\tau_H = C_{gs} R_{gs} + (C_{gd} + C_L) R_{gd} = 20 \times 10^{-15} \times 0.93 \times 10^3 + (5 + 25) \times 10^{-15} \times 19.5 \times 10^3 = 18.6 \times 10^{-12} + 585 \times 10^{-12} = 603.6\text{ps}$$

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi \times 603.6 \times 10^{-12}} = 263.7\text{MHz}$$

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