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EE115 Analog Circuits pn Junction

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Outline

- Semiconductors
 - pn Junction
 - pn Junction with an Applied Voltage
 - Capacitive Effects in the pn Junction
- Reading: SEDTRA/SMITH book pages 150-169

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pn Junction

- P-type in contact with n-type
- Basic building blocks of semiconductor devices
 - Diodes
 - Bipolar junction transistors (BJT)
 - Metal-oxide-semiconductor field effect transistors (MOSFET)

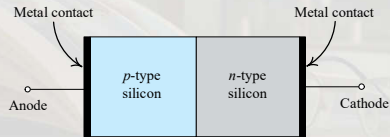


Fig. Simplified physical structure of the pn junction.

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Built-in Voltage

- At pn junction, free electrons from n-side **recombine** with free holes from p-side.
- **Depletion region** has no electrons or holes, but has fixed (immobile) charges from donor and acceptor ions
- The fixed charges establish an **electric field**, and create a potential difference between p and n-sides.

$$V_0 = V_T \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

- This potential is called **built-in potential**
 - V_T : thermal voltage (≈ 26 mV at room temp)
 - N_A : acceptor concentration on p-side
 - N_D : donor concentration on n-side
 - n_i : intrinsic carrier concentration ($\approx 1.5 \times 10^{10} \text{ cm}^{-3}$ at room temp)

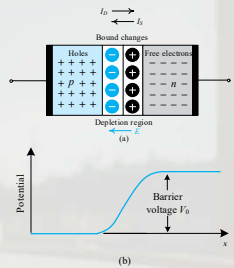
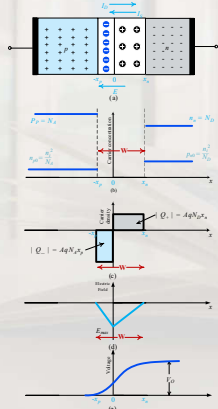


Fig. (a) pn junction with zero bias. (b) Potential distribution along an axis perpendicular to the junction.

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Electrostatic Analysis of pn Junction



Gauss' s Law: The net **electric flux** through any closed surface is equal to $1/\epsilon$ times the net electric charge within that closed surface

$$\text{Charge density: } q(x) = \begin{cases} -qN_A, & -x_p < x < 0 \\ qN_D, & 0 < x < x_n \end{cases}$$

Gauss Law: $\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_s}$, where $\epsilon_s = 11.7\epsilon_0$, $\epsilon_0 = 8.854 \times 10^{-14} \text{F/cm}$

In one dimension:

$$E(x)A - E(-x_p)A = \frac{-qN_A(x + x_p)}{\epsilon_s}$$

$$E(-x_p) = 0 \text{ (in charge neutral region, } N_A = p)$$

$$E(x) = \begin{cases} -\frac{qN_A(x + x_p)}{\epsilon_s}, & -x_p < x < 0 \\ \frac{qN_D(x - x_n)}{\epsilon_s}, & 0 < x < x_n \end{cases}$$

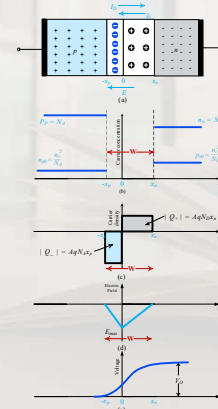
Note: $N_A x_p = N_D x_n$ (charge equality)

Maximum electrical field occurs at $x = 0$ (junction)

$$E_{\max} = -\frac{qN_A x_p}{\epsilon_s} = -\frac{qN_D x_n}{\epsilon_s}$$

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Continued 1



$$E(x) = \begin{cases} -\frac{qN_A(x + x_p)}{\epsilon_s}, & -x_p < x < 0 \\ \frac{qN_D(x - x_n)}{\epsilon_s}, & 0 < x < x_n \end{cases}$$

$$V(x) = -\int_{-x_p}^x E(x') dx'$$

$$V(x) = \begin{cases} \frac{qN_A}{2\epsilon_s} (x + x_p)^2, & -x_p < x < 0 \\ \frac{q}{2\epsilon_s} (N_A x_p^2 + N_D x_n^2) - \frac{qN_D}{2\epsilon_s} (x - x_n)^2, & 0 < x < x_n \end{cases}$$

$$V(x_n) = \frac{q}{2\epsilon_s} (N_A x_p^2 + N_D x_n^2) = V_o^*$$

$$x_p = \frac{N_D}{N_A + N_D} W, \quad x_n = \frac{N_A}{N_A + N_D} W$$

Insert into *, solve W:

$$W = \sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) V_o} \quad \text{Depletion region width}$$

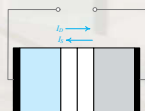
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Depletion Width Under Bias

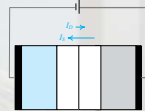
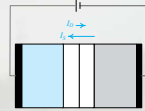


$$W = \sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) (V_o - V)}$$

V is the applied voltage to the pn junction, it is **positive** for **forward** bias and **negative** for **reverse** bias. Depletion width is widened in reverse bias



(a) equilibrium;

(b) reverse bias;
Fig. The pn junction in:

(c) forward bias

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Current-Voltage (I-V) Characteristics

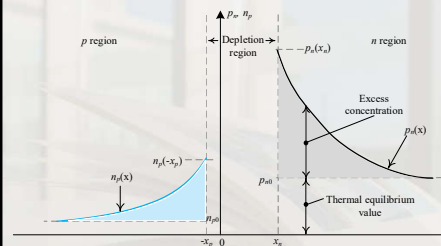


Fig. Minority-carrier distribution in a forward-biased pn junction. Suppose $N_A \gg N_D$

Under **forward bias**, minority carriers at the edge of the depletion region is boosted up by e^{V/V_T} times.

$$\text{Excess concentration} = p_{n0}(e^{V/V_T} - 1)$$

$$p_n(x) = p_{n0} + p_{n0}(e^{V/V_T} - 1) \cdot e^{-\frac{x-x_n}{L_p}}$$

L_p : hole diffusion length in n-type

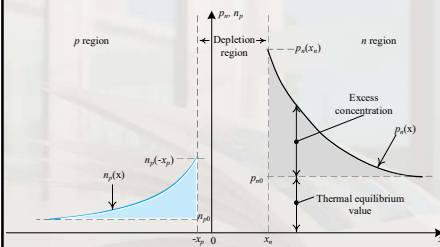
Hole diffusion current density on n-side

$$J_p(x_n) = -qD_p \left. \frac{dp_n(x)}{dx} \right|_{x=x_n} = q \frac{D_p}{L_p} p_{n0} (e^{V/V_T} - 1)$$



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Continued



Similarly, electron diffusion current density in p-side

$$J_n(-x_p) = -qD_n \left. \frac{dn_p(x)}{dx} \right|_{x=-x_p} = q \frac{D_n}{L_n} n_{p0} (e^{V/V_T} - 1)$$

Total current:

$$I = A(J_p + J_n) = A \left(q \frac{D_p}{L_p} p_{n0} + q \frac{D_n}{L_n} n_{p0} \right) (e^{V/V_T} - 1)$$

$$= Aq n_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right) (e^{V/V_T} - 1)$$

Thus,

$$I = I_s (e^{V/V_T} - 1)$$

Where,

$$I_s = Aq n_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right)$$



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I-V Curve

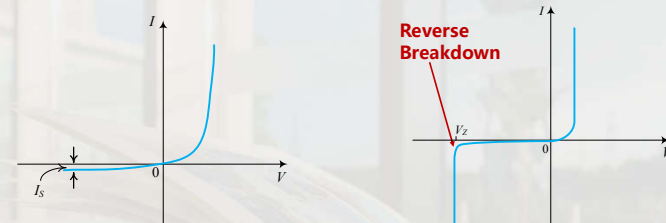


Fig. pn junction I-V characteristic.

$$I = I_s (e^{V/V_T} - 1)$$

$$I_s = Aq n_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right)$$

Reverse
Breakdown

Fig. I-V characteristic of the pn junction with breakdown region.

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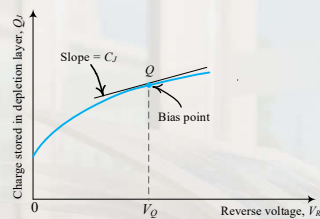
Depletion Capacitance (mainly **reverse bias**)

Fig. Charge stored on either side of the depletion layer as a function of V_R .

If $V = -V_R$, junction area A

Total charge in depletion region,

$$Q_J = AqN_D x_n = AqN_D \frac{N_A}{N_A + N_D} W$$

$$W = \sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 + V_R)}$$

As **bias voltage** change, **junction charge** change.

This is a **nonlinear** capacitor.

The capacitance is

$$C_J = \frac{dQ_J}{dV_R} = Aq \frac{N_A N_D}{N_A + N_D} \sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right)} \frac{d}{dV} \sqrt{(V_0 + V_R)}$$

$$C_J = A \sqrt{\frac{\epsilon_s q}{2} \left(\frac{N_A N_D}{N_A + N_D} \right)} \frac{1}{\sqrt{(V_0 + V_R)}}$$

Note:

$$C_J = \frac{\epsilon_s A}{W}$$



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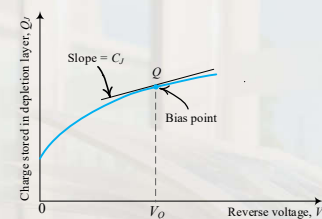


Fig. Charge stored on either side of the depletion layer as a function of V_R .

At zero bias, $V_R = 0$,

$$C_{J0} = A \sqrt{\frac{\epsilon_s q}{2} \left(\frac{N_A N_D}{N_A + N_D} \right)} \frac{1}{\sqrt{V_0}}$$

Therefore at $V = -V_R$

$$C_J = \frac{C_{J0}}{\sqrt{1 + V_R/V_0}}$$

This is a variable capacitor, controlled by voltage!

In comparison, for a **linear** capacitor:

$$C = \frac{Q}{V}$$

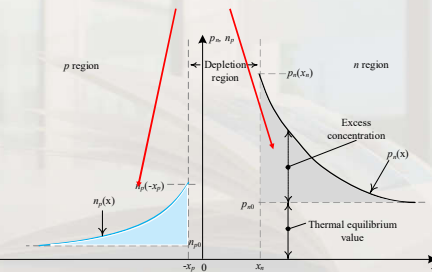
C is a **constant**.

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Diffusion Capacitance (**forward bias**)



Extra minority carriers stored outside junction under forward bias



$$Q_p = Aq \times \text{shaded area under } p_n(x) \text{ curve}$$

$$\text{Excess concentration} = p_{n0}(e^{V/V_T} - 1) \cdot e^{-\frac{x-x_n}{L_p}}$$

$$Q_p = Aq \int_{x_n}^{\infty} p_{n0}(e^{V/V_T} - 1) \cdot e^{-\frac{x-x_n}{L_p}} dx$$

$$Q_p = Aq L_p p_{n0}(e^{V/V_T} - 1) = \frac{L_p^2}{D_p} I_p = \tau_p I_p$$

τ_p has unit of time, its physical meaning is

Minority carrier lifetime:

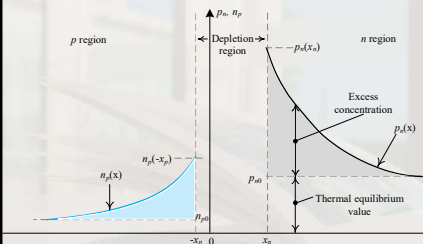
$$\tau_p = \frac{L_p^2}{D_p}$$

Average time it takes for a hole injected into n region to recombine with a majority electron.

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Similarly, $Q_n = \tau_n I_n$

Total charge stored: $Q = \tau_p I_p + \tau_n I_n = \tau_T I$

Where, τ_T is the mean transit time.

These stored charges correspond to another nonlinear capacitor called **diffusion capacitance**

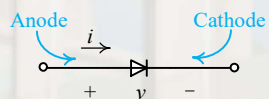
$$C_d = \frac{dQ}{dV} = \frac{d(\tau_T I)}{dV} = \tau_T \frac{dI}{dV}$$

Hence,

$$C_d = \left(\frac{\tau_T}{V_T} \right) I$$

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Summary of pn Junction



$$\text{Built-in potential: } V_0 = V_T \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

$$\text{Depletion Width: } W = \sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 - V)}$$

Forward bias

$$\text{I-V curve: } I = I_s (e^{V/V_T} - 1)$$

$$\text{Diffusion capacitance: } C_d = \left(\frac{\tau_T}{V_T} \right) I$$

Reverse Bias

Negligible current, $I = -I_s$

$$\text{Depletion capacitance: } C_j = \frac{C_{j0}}{\sqrt{1 + V_R/V_0}}$$

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