上海科技大学
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# EE115 Analog Circuits
# Diff Amp: CMRR, offset, current mirror load

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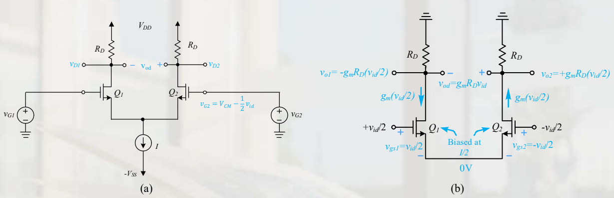
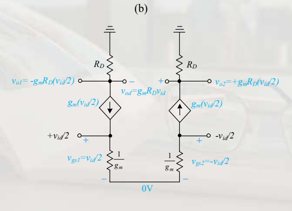
## Outline

- Differential amplifiers 2
  - Common-Mode Rejection
  - DC Offset
  - Differential Amplifier with Current Mirror Load
- Reading: SEDTRA/SMITH book pages 609-639



## Review: Small Signal Operation




$$v_{G1} = V_{CM} + \frac{1}{2} v_{id}$$

$$v_{G2} = V_{CM} - \frac{1}{2} v_{id}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

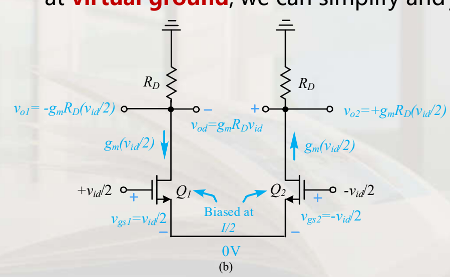
$$A_d \equiv \frac{v_{od}}{v_{id}} = \frac{v_{o2} - v_{o1}}{v_{id}} = g_m R_D$$

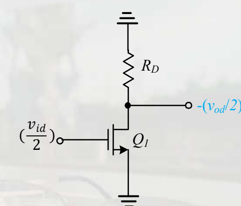
- For **differential AC small signal**, the differential pair is **anti-symmetric**. The potential at the mid point is zero. This is called **Virtual Ground**
- This virtual ground is obtained **without bypass capacitor** -> much smaller area and better frequency response



## Review: Differential Half Circuit

- Because the two halves of the circuits are **anti-symmetric**, and **source** is at **virtual ground**, we can simplify and just analyze the **half circuit**

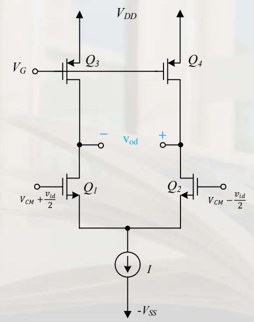


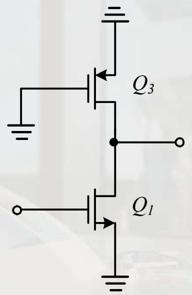



Equivalent half circuit
If consider $r_o$:

$$A_d = g_m (R_D || r_o)$$

## Review: Diff. Amplifier w/ Current-Source Loads



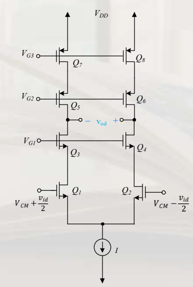

$$A_d \equiv \frac{v_{od}}{v_{id}} = g_{m1}(r_{o1}||r_{o3})$$



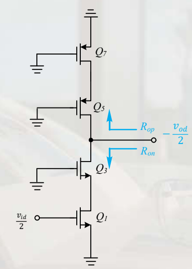


## Review: Cascode Differential Amplifier

- Cascode configurations for both amplifying transistors and current source loads.



(a)



(b)

$$A_d \equiv \frac{v_{od}}{v_{id}} = g_{m1}(R_{on}||R_{op})$$

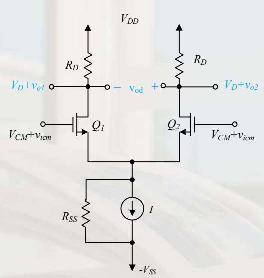
$$R_{on} = (g_{m3}r_{o3})r_{o1}$$

$$R_{op} \equiv (g_{m5}r_{o5})r_{o7}$$

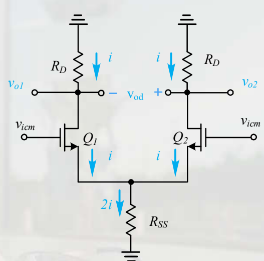




## AC Equivalent Circuit for Common Mode Input




(a)




(b)

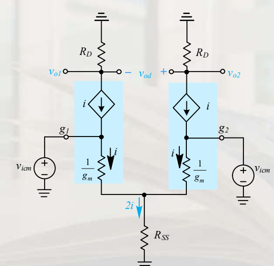
- **Non-ideal** current source consists of an ideal current source, shunted by a large resistance, $R_{ss}$.



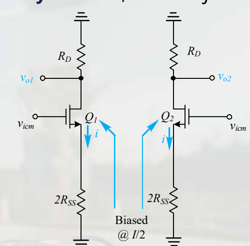


## Common Mode Half Circuit

- For **differential inputs**, the two half circuits are **anti-symmetric**, and the joint (Source) is always at **virtual ground**.




$v_{icm}$: ac small signal common-mode input

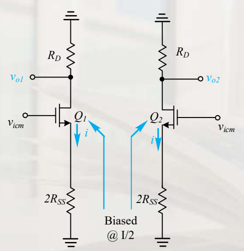


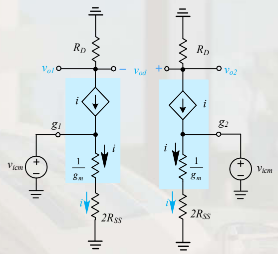

- For small signal **common-mode** inputs, the two half circuits are **symmetric**. The Source is **not virtual ground** any more.
- $R_{SS}$ can be considered as two parallel $2R_{SS}$.
- Each CM half circuit has $2R_{SS}$ connected to the source.

## Ideal CM Output Voltage



**Common-Source with degeneration**




$$v_{icm} = \frac{i}{g_m} + 2iR_{SS}$$

$$i = \frac{v_{icm}}{1/g_m + 2R_{SS}}$$

$$v_{o1} = v_{o2} = -\frac{R_D}{1/g_m + 2R_{SS}} v_{icm}$$

$$\frac{v_{o1}}{v_{icm}} = \frac{v_{o2}}{v_{icm}} \cong -\frac{R_D}{2R_{SS}}$$

$$v_{od} = v_{o1} - v_{o2} = 0$$

- $v_{od}$ is **0** for **ideal** diff pair
  - 1. Perfectly matched transistors and resistors.
  - 2. Small CM voltage to keep $Q_1$&$Q_2$ in saturation.

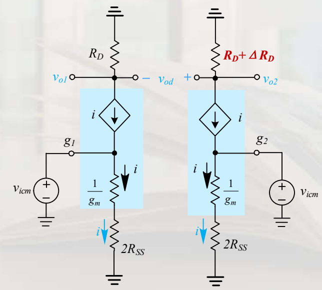


## Common Mode Gain with Mismatched $R_D$



- However, any **mismatch** in the half circuits will produce **finite output voltage**.

**Case 1: $R_D$ mismatch**




$$v_{o1} \cong -\frac{R_D}{2R_{SS}} v_{icm}$$

$$\frac{v_{o1}}{v_{icm}} = \frac{v_{o2}}{v_{icm}} \cong -\frac{R_D}{2R_{SS}}$$

$$v_{o2} \cong -\frac{R_D + \Delta R_D}{2R_{SS}} v_{icm}$$

$$v_{od} = v_{o1} - v_{o2} = -\frac{\Delta R_D}{2R_{SS}} v_{icm}$$

$$A_{cm} \equiv \frac{v_{od}}{v_{icm}} = -\frac{\Delta R_D}{2R_{SS}} = -\left(\frac{R_D}{2R_{SS}}\right)\left(\frac{\Delta R_D}{R_D}\right)$$

$$\text{CMRR}(dB) = 20\log\frac{|A_d|}{|A_{cm}|}$$

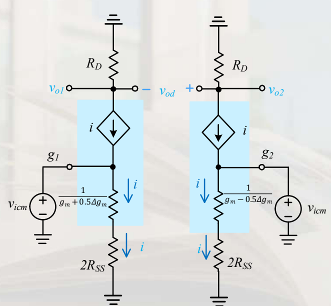
Since $A_d = g_m R_D$, $\quad \text{CMRR} = (2g_m R_{SS})/\left(\frac{\Delta R_D}{R_D}\right)$



## Common Mode Gain with Mismatched $g_m$



**Case 2: $g_m$ mismatch**




$$g_{m1} = g_m + \frac{1}{2}\Delta g_m$$

$$g_{m2} = g_m - \frac{1}{2}\Delta g_m$$

$$g_{m1} - g_{m2} = \Delta g_m$$

$$A_{cm} \cong \left(\frac{R_D}{2R_{SS}}\right)\left(\frac{\Delta g_m}{g_m}\right)$$

$$\text{CMRR} = (2g_m R_{SS})/\left(\frac{\Delta g_m}{g_m}\right)$$

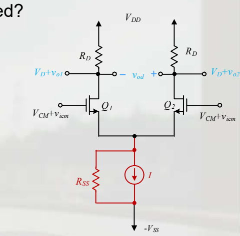


## Example-CMRR of MOS differential pair



- Design a MOS diff pair with 100 dB CMRR. The only source of mismatch is a 2% mismatch in $Q_1$ and $Q_2$'s W/L ratios. Let $I = 200\mu A$ and assume that all transistors have $V_{OV} = 0.2V$. $V_A' = 5A/\mu m$.
  - a) If a **simple current source** is used for $I$, what channel length is needed?

**Solution:**




Mismatch of W/L ratios leads to mismatch of $g_m$

$$g_m = (\mu_n C_{ox})\left(\frac{W}{L}\right)V_{OV}$$

$$\text{CMRR} = (2g_m R_{SS})/\left(\frac{\Delta g_m}{g_m}\right)$$

$$100dB = 10^5 = \frac{(2g_m R_{SS})}{0.02} *$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times (I/2)}{V_{OV}} = \frac{2 \times 0.1}{0.2} = 1mA/V$$

Substituting into *,

$$R_{SS} = 1M\Omega$$

For a **simple current source**

$$r_o = R_{SS} = 1M\Omega$$

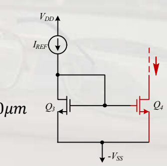
$$r_o = \frac{V_A}{I} = 1M\Omega$$

Since $I = 200\mu A$

$$V_A = 200V$$

$$V_A = V_A' L = 5L \rightarrow L = 40\mu m$$

**Too large!**



# Example-continued



➢ b) If a **cascode current source** is used for $I$, what channel length is needed?

**Solution:**

For a **cascode current source**

$$R_{SS} = (g_m r_o) r_o = 1M\Omega$$

$$g_m = \frac{2I}{V_{OV}} = \frac{2 \times 0.2}{0.2} = 2mA/V$$

Thus $1M\Omega = 2mA/V \times r_o^2$

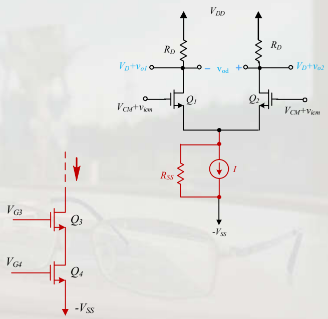
$$r_o = 22.36k\Omega$$

$$r_o = 22.36k\Omega = \frac{V_A}{I} = \frac{V_A}{0.2}$$

$$V_A = 4.47V = V_A' L = 5L$$

$$L = 0.89\mu m$$

**A considerable reduction of chip area!**

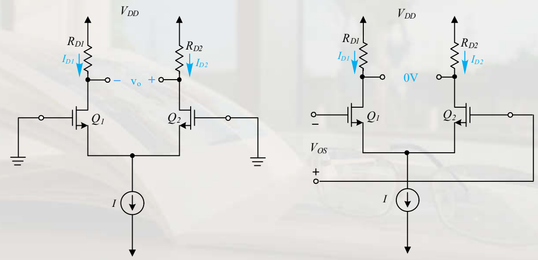





# DC Offset



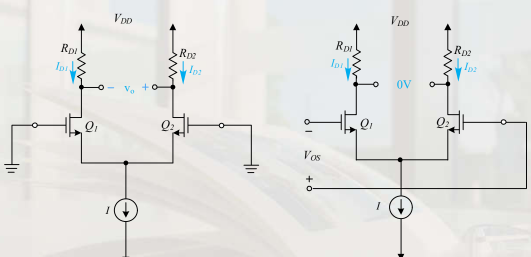
- Due to mismatch in $R_D$, $V_o \neq 0$ even when both inputs are grounded.
- To produce 0 output, an **input offset voltage** $V_{OS} = V_O / A_d$ needs to be applied. $A_d$ is differential gain






# DC Offset






$$R_{D1} = R_D + \frac{\Delta R_D}{2}$$

$$R_{D2} = R_D - \frac{\Delta R_D}{2}$$

$$V_{D1} = V_{DD} - \frac{I}{2}\left(R_D + \frac{\Delta R_D}{2}\right)$$

$$V_{D2} = V_{DD} - \frac{I}{2}\left(R_D - \frac{\Delta R_D}{2}\right)$$

$$V_o = V_{D2} - V_{D1} = \left(\frac{I}{2}\right)\Delta R_D$$

$$V_{OS} = \left(\frac{V_{OV}}{2}\right)\left(\frac{\Delta R_D}{R_D}\right)$$

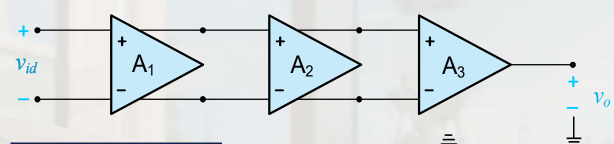
$$A_d \equiv \frac{v_{od}}{v_{id}} = \frac{v_{o2} - v_{o1}}{v_{id}} = g_m R_D$$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{I}{V_{OV}}$$

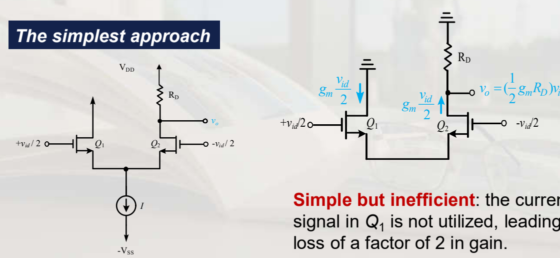


# Differential-to-Single-Ended Conversion






**The simplest approach**




**Simple but inefficient**: the current signal in $Q_1$ is not utilized, leading to loss of a factor of 2 in gain.

# Differential Pair w/ Current Mirror Load

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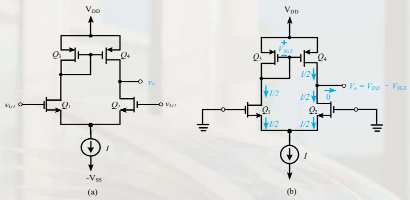
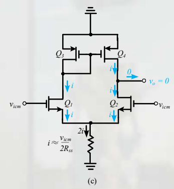
**A much better approach**

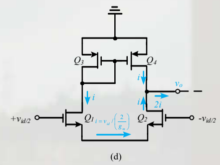


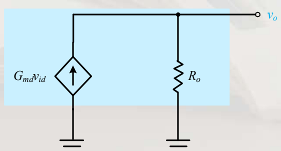

Common mode input




Differential input

**Output current 2*i*: No loss of gain!**




$$G_{md} = g_{m1,2}$$

$$R_o = r_{o2}||r_{o4} = R_{on}||R_{op}$$

$$A_d \equiv \frac{v_o}{v_{id}} = G_{md}R_o = g_{m1,2}(r_{o2}||r_{o4})$$

$$A_d = g_{m1,2}(R_{on}||R_{op})$$

$$A_d = \frac{1}{2}g_m r_o = \frac{1}{2}A_o$$