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EE115 Analog Circuits
Frequency Response

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Outline

- High Frequency Response 2
  - Frequency Response of Common-Source Amplifiers
  - Open-Circuit Time Constants (OCTC) Method
  - High Frequency Response of Common Gate Amplifiers
- Reading: SEDTRA/SMITH book pages 686-715

Review: Discrete vs Integrated Circuits

Discrete Circuits

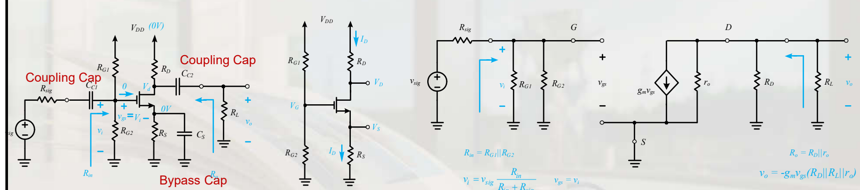
- Resistors and capacitors are frequently used
- AC coupled with **coupling capacitors**
- High DC power supply voltage
- Transistor choice limited to available parts
- Mostly BJT, some MOS

IC

- Use mostly transistors
  - Resistors and capacitors occupy **too much areas**
- Mostly **DC coupled** (without capacitors)
- Low DC power supply voltage (~1V)
- Can vary device size
- Predominantly CMOS
  - BiCMOS provides BJT

Review: CS Amplifier with Bias Circuit

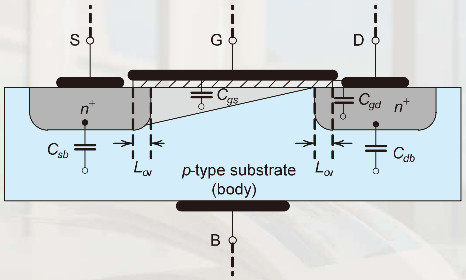
- Both coupling and bypass capacitors are DC-open and AC-short



$$R_{in} = R_{G1}||R_{G2}$$

$$G_v = -\frac{R_{in}}{R_{in} + R_{sig}} g_m (R_D||R_L||r_o)$$

Review: Cross Section of MOSFET w/ Internal Cap.



- Gate/Source capacitance:

$$C_{gs} = \frac{2}{3} W L C_{ox} + C_{ov}$$

where

- $C_{ox} = \varepsilon_{ox}/t_{ox}$ $[F/cm^2]$
- W: Transistor width
- L: Gate Length
- $C_{ov} = W L_{ov} C_{ox}$
- L_{ov}: overlap between Gate/Source or Gate/Drain, typically 0.05-0.1L

- Gate/Drain capacitance:

$$C_{gd} = C_{ov}$$

- MOSFET has several internal capacitances, which take time to charge/discharge, limiting the transistor speed.

Review: MOSFET High-f Equivalent-Circuit Model

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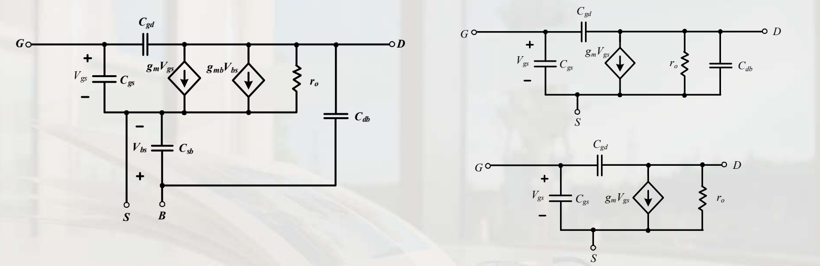

- Source is connected to the body (no **body effect**)
- Capacitance between drain and body C_{db} neglected

Review: Unity-Gain Frequency, f_T

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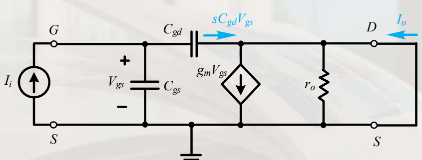
- f_T: frequency at which short-circuit current gain =1
  - FoM for transistor speed
  - Drain is grounded (short-circuit load)



$$I_o = g_m V_{gs} - sC_{gd} V_{gs} = (g_m - sC_{gd}) V_{gs}$$

$$V_{gs} = \frac{I_i}{s(C_{gs} + C_{gd})}$$

Combine: $$\frac{I_o}{I_i} = \frac{g_m - sC_{gd}}{s(C_{gs} + C_{gd})}$$

$$\left|\frac{I_o}{I_i}\right|(s = j\omega_T) \cong \frac{g_m}{\omega_T(C_{gs} + C_{gd})} = 1$$

Unity gain freq.: $$\omega_T = \frac{g_m}{(C_{gs} + C_{gd})}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

As gate length reduces in advanced technology, C_{gs} reduces and f_T increases

High Frequency Response of CS Amplifier

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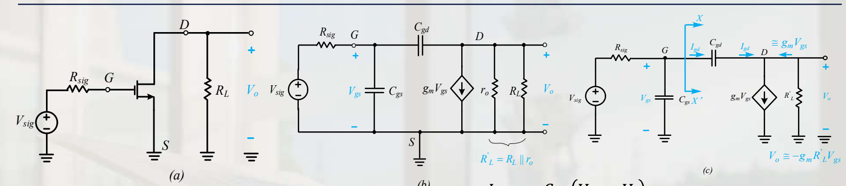


- Middle band gain, A_M
  - C_{gd}, C_{gs} ignored

$$R'_L = R_L \| r_o \qquad A_M = \frac{V_o}{V_{sig}} = -g_m R'_L$$

$$I_{gd} = sC_{gd}(V_{gs} - V_o)$$

$$= sC_{gd}[V_{gs} - (-g_m R'_L V_{gs})]$$

$$= \mathbf{sC_{gd}(1 + g_m R'_L) V_{gs}}$$

$$V_o = -(g_m V_{gs} - I_{gd}) R'_L$$

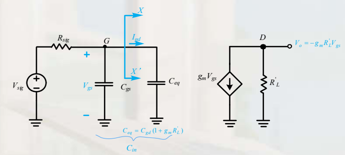
Middle band

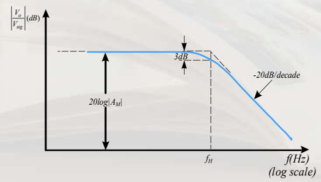
$$\frac{V_o}{V_{gs}} = -[g_m - sC_{gd}(1 - g_m R'_L)] R'_L \cong -g_m R'_L$$

Tedious, not intuitive.

Miller Capacitance

Equivalent Circuit





$$I_{gd} = \mathbf{sC_{gd}(1+g_m R_L')}V_{gs} = sC_{eq}V_{gs}$$

$$C_{eq} = C_{gd}(1+g_m R_L')$$

Miller effect, $(1+g_m R_L')$ is known as Miller multiplier

$$V_{gs} = \frac{1}{1+\frac{s}{\omega_p}}V_{sig}$$

Where, ω_p is the pole frequency of the STC circuit

$$\omega_P = 1/(C_{in}R_{sig}),\ C_{in} = C_{gs} + C_{eq} = C_{gs} + C_{gd}(1+g_m R_L')$$

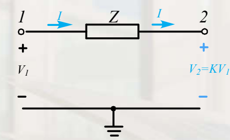
$$\frac{V_o}{V_{sig}} = -\frac{g_m R_L'}{1+\frac{s}{\omega_P}} = \frac{\mathbf{A_M}}{\mathbf{1+\frac{s}{\omega_H}}}$$

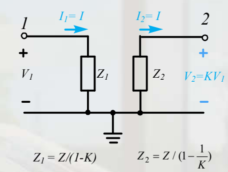
$$\omega_H = \omega_P = 1/(C_{in}R_{sig})$$

- Large input capacitance in CS amplifier due to Miller effect, which greatly reduced the bandwidth of CS amp

Miller's Theorem

Miller Equivalent





If $V_2 = KV_1$

$$I = \frac{V_1 - KV_1}{Z}$$

Input side:

$$I_1 = \frac{V_1}{Z_1} = I = \frac{V_1 - KV_1}{Z}$$

$$Z_1 = \frac{Z}{(1-K)}$$

Output side:

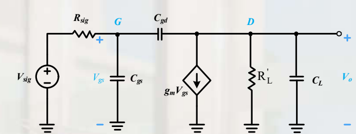
$$I_2 = \frac{0-V_2}{Z_2} = \frac{0-KV_1}{Z_2} = I = \frac{V_1 - KV_1}{Z}$$

$$Z_2 = \frac{Z}{\left(1-\frac{1}{K}\right)}$$

Open-Circuit Time Constant (OCTC) Method for High Cut-off Frequency

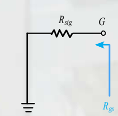
- Replace all **capacitors** by **open circuit**
- **Signal source** becomes **zero**
- Consider one capacitor at a time, find resistance R_i **seen** by the i-th capacitor, C_i
- $\omega_H \approx \frac{1}{\sum_i C_i R_i}$

Applying OCTC to CS Amplifier



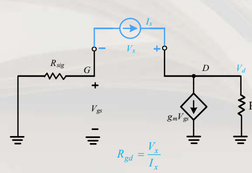
Capacitor = Open **Signal source = 0**

1. Only consider C_{gs}, find R_{gs}



$$R_{gs} = R_{sig} \quad \cdots (1)$$

2. Only consider C_{gd}, find R_{gd}



$$V_{gs} = -I_x R_{sig}$$

$$V_d = V_x + V_{gs}$$

$$I_x = g_m V_{gs} + \frac{V_d}{R_L'}$$

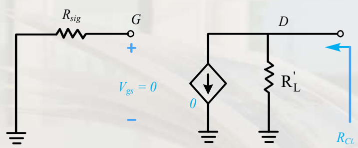
$$I_x = g_m V_{gs} + \frac{V_x + V_{gs}}{R_L'}$$

$$R_{gd} \equiv \frac{V_x}{I_x} = R_{sig}(1+g_m R_L') + R_L' \cdots (2)$$

Applying OCTC to CS Amplifier

$R_{gs} = R_{sig} \cdots (1)$

$R_{gd} = R_{sig}(1 + g_m R_L') + R_L' \cdots (2)$



Capacitor = Open | **Signal source = 0**

3. Only consider C_L, find R_{CL}

$R_{C_L} = R_L' \cdots (3)$

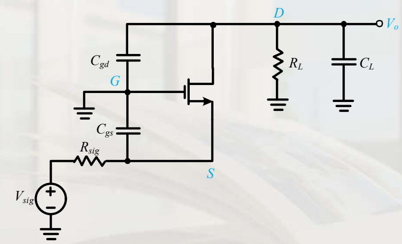
$$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{C_L}$$
$$= C_{gs}R_{sig} + C_{gd}\left[R_{sig}(1 + g_m R_L') + R_L'\right] + C_L R_L'$$

$$f_H = \frac{1}{2\pi\tau_H}$$

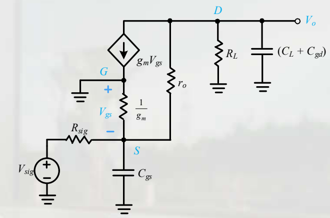
$$\tau_H = \left[C_{gs} + C_{gd}(1 + g_m R_L')\right]R_{sig} + (C_{gd} + C_L)R_L'$$

- Time constant from **input** port of Miller Equivalent Circuit
- Time constant from **output** port of Miller Equivalent Circuit

High f Response of Common Gate Amplifier

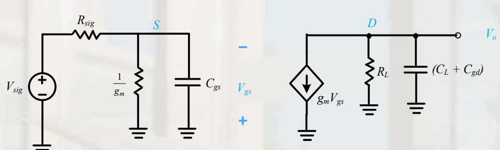


(a) CG amplifier with cap.(ac equivalent)



(b) Equivalent ac small signal model

High f Response of Discrete Circuit CG Amplifier



Equivalent circuit with r_o neglected

$$f_{p1} = \frac{1}{2\pi C_{gs}\left(R_{sig}||\frac{1}{g_m}\right)}$$

$$\tau_{gs} = C_{gs}\left(R_{sig}||\frac{1}{g_m}\right) = \frac{1}{2\pi f_{p1}}$$

$$f_{p2} = \frac{1}{2\pi(C_{gd} + C_L)R_L}$$

$$\tau_{gd} = (C_L + C_{gd})R_L = \frac{1}{2\pi f_{p2}}$$

$$\tau_H = C_{gs}\left(R_{sig}||\frac{1}{g_m}\right) + (C_L + C_{gd})R_L$$

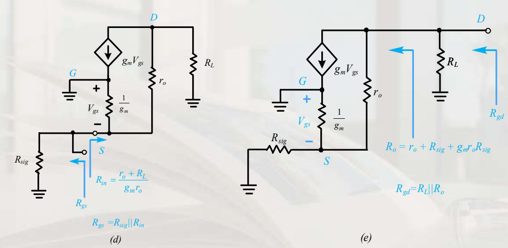
$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{\frac{1}{f_{p1}} + \frac{1}{f_{p2}}}$$

- Discrete circuit CG: early effect can be ignored
- **No Miller effect** since both capacitance are grounded
- The dominant term is likely to be $C_{gs}\left(R_{sig}||\frac{1}{g_m}\right)$, which is small → High f_H
  - Common-Gate is a **broadband** amplifier

Why early effect can be ignored in discrete circuit?

High f Response of IC CG Amplifier

- IC CG: early effect must be **considered**.
- Apply **OCTC** method.



Capacitor = Open | **Signal source = 0**

$$R_{gs} = R_{sig}||\ R_{in}$$

$$R_{in} = \frac{r_o + R_L}{1 + g_m r_o} \cong \frac{r_o + R_L}{g_m r_o}$$

$$R_{gd} = R_L||\ R_o$$

$$R_o = r_o + R_{sig} + g_m r_o R_{sig}$$

$$\tau_H = \tau_{gs} + \tau_{gd}$$

$$f_H = \frac{1}{2\pi\tau_H}$$

Example: frequency response of CG amplifier

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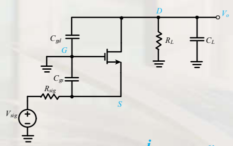
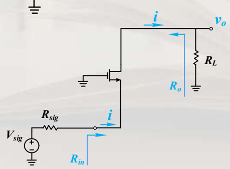
Consider a CG amplifier with $g_m = 2mA/V$, $r_o = 20k\Omega$, $C_{gs} = 20fF$, $C_{gd} = 5fF$, $C_L = 25fF$, $R_{sig} = 20k\Omega$, and $R_L = 20k\Omega$. Determine the input resistance, the midband gain, and the upper 3-dB frequency f_H





Midband equivalent

Solution:

$v_o = iR_L$

$v_{sig} = i(R_{sig} + R_{in})$

$$G_v = \frac{v_o}{v_{sig}} = \frac{R_L}{R_{sig} + R_{in}}$$

$$R_{in} = \frac{r_o + R_L}{1 + g_m r_o}$$

$$= \frac{20+20}{1+(2\times 20)} = 0.98k\Omega$$

$$G_v = \frac{20}{20 + 0.98} = \frac{0.95V}{V}$$

$R_{gs} = R_{sig}||R_{in}$

$= 20||0.98 = 0.93k\Omega$

$R_{gd} = R_L||R_o$

$R_o = r_o + R_{sig} + (g_m r_o)R_{sig}$

$= 20 + 20 + 40 \times 20 = 840k\Omega$

$R_{gd} = 20||840 = 19.5k\Omega$

$\tau_H = C_{gs}R_{gs} + (C_{gd} + C_L)R_{gd}$

$= 20 \times 10^{-15} \times 0.93 \times 10^3 + (5 + 25)$
$\times 10^{-15} \times 19.5 \times 10^3$
$= 18.6 \times 10^{-12} + 585 \times 10^{-12} = 603.6ps$

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 603.6 \times 10^{-12}} = 263.7MHz$$