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EE115 Analog Circuits
pn Junction

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Outline

- Semiconductors
  - pn Junction
  - pn Junction with an Applied Voltage
  - Capacitive Effects in the pn Junction
- Reading: SEDTRA/SMITH book pages 150-169

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pn Junction

- P-type in contact with n-type
- Basic building blocks of semiconductor devices
  - Diodes
  - Bipolar junction transistors (BJT)
  - Metal-oxide-semiconductor field effect transistors (MOSFET)

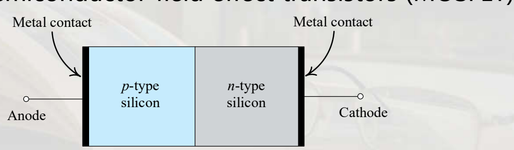

Fig. Simplified physical structure of the *pn* junction.

Built-in Voltage

- At pn junction, free electrons from n-side **recombine** with free holes from p-side.
- **Depletion region** has no electrons or holes, but has fixed (immobile) charges from donor and acceptor ions
- The fixed charges establish an **electric field**, and create a potential difference between p and n-sides.

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

- This potential is called **built-in potential**
  - V_T: thermal voltage (= 26 mV at room temp)
  - N_A : acceptor concentration on p-side
  - N_D : donor concentration on n-side
  - n_i : intrinsic carrier concentration (=1.5×10^{10} cm^{-3} at room temp)

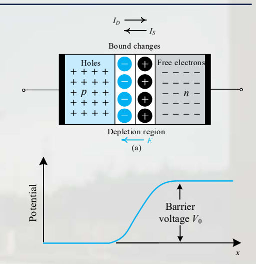

Fig. **(a)** *pn* junction with zero bias. **(b)** Potential distribution along an axis perpendicular to the junction.

Electrostatic Analysis of pn Junction

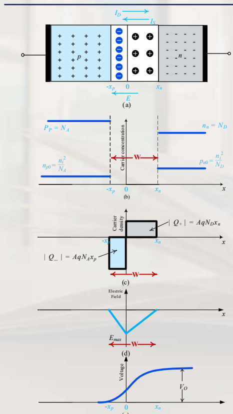

Fig. A pn junction with the terminals open-circuited, note $N_A > N_D$

Gauss' s Law: The net ***electric flux*** through any closed surface is equal to 1/ε times the net electric charge within that closed surface

Charge density: $q(x) = \begin{cases} -qAN_A, -x_p < x < 0 \\ qAN_D, \ 0 < x < x_n \end{cases}$

Gauss Law: $\oint \vec{E} \cdot \vec{dA} = \frac{Q}{\varepsilon_s}$, where $\varepsilon_s = 11.7\varepsilon_o$, $\varepsilon_o = 8.854 \times 10^{-14}$F/cm

In one dimension:

$$E(x)A - E(-x_p)A = \frac{-qAN_A(x + x_p)}{\varepsilon_s}$$

$E(-x_p) = 0$ (in charge neutral region, $N_A = p$)

$$E(x) = \begin{cases} \frac{-qN_A(x + x_p)}{\varepsilon_s}, & -x_p < x < 0 \\ \frac{qN_D(x - x_n)}{\varepsilon_s}, & 0 < x < x_n \end{cases}$$

Note: $N_A x_p = N_D x_n$(charge equality)

Maximum electrical field occurs at x = 0 (junction)

$$E_{max} = -\frac{qN_A x_p}{\varepsilon_s} = -\frac{qN_D x_n}{\varepsilon_s}$$

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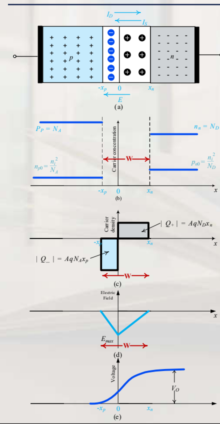


$$E(x) = \begin{cases} \frac{-qN_A(x + x_p)}{\varepsilon_s}, & -x_p < x < 0 \\ \frac{qN_D(x - x_n)}{\varepsilon_s}, & 0 < x < x_n \end{cases}$$

$$V(x) = -\int_{-x_p}^{x} E(x')dx'$$

$$V(x) = \begin{cases} \frac{qN_A}{2\varepsilon_s}(x + x_p)^2, & -x_p < x < 0 \\ \frac{q}{2\varepsilon_s}(N_A x_p^2 + N_D x_n^2) - \frac{qN_D}{2\varepsilon_s}(x - x_n)^2, & 0 < x < x_n \end{cases}$$

$V(x_n) = \frac{q}{2\varepsilon_s}(N_A x_p^2 + N_D x_n^2) = V_o$ *

$x_p = \frac{N_D}{N_A + N_D}W$, $x_n = \frac{N_A}{N_A + N_D}W$

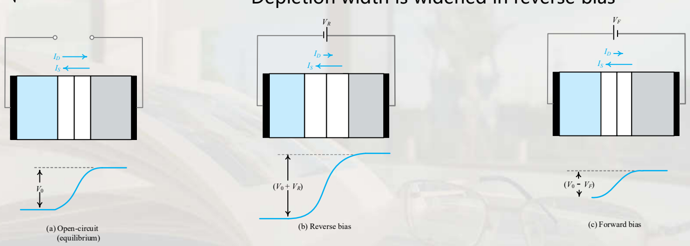
Insert into *, solve W:

$W = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_0}$ **Depletion region width**

Depletion Width Under Bias

$$W = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_o - V)}$$

V is the applied voltage to the pn junction,
it is **positive** for **forward** bias and **negative** for **reverse** bias.
Depletion width is widened in reverse bias



(a) equilibrium; (b) reverse bias; (c) forward bias
Fig. The *pn* junction in:

Current-Voltage (I-V) Characteristics

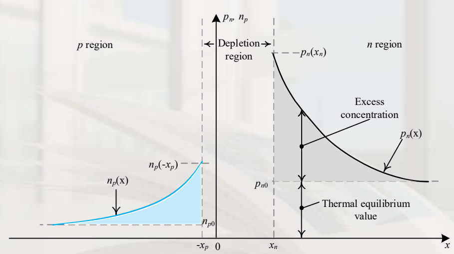


Fig. Minority-carrier distribution in a forward-biased pn junction. Suppose $N_A >> N_D$

Under **forward bias**, minority carriers at the edge of the depletion region is boosted up by e^{V/V_T} times.

Excess concentration = $p_{n0}(e^{V/V_T} - 1)$

$$p_n(x) = p_{n0} + p_{n0}(e^{V/V_T} - 1) \cdot e^{-\frac{x - x_n}{L_p}}$$

L_p: hole diffusion length in n-type

Hole diffusion current density on n-side

$$J_p(x_n) = -qD_p \left.\frac{dp_n(x)}{dx}\right|_{x=x_n} = q\frac{D_p}{L_p}p_{n0}(e^{V/V_T} - 1)$$

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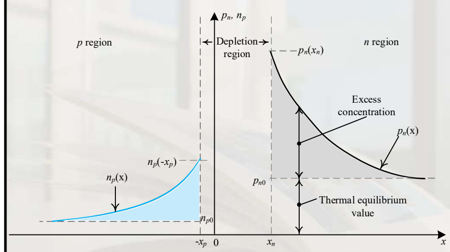

Similarly, electron diffusion current density in p-side

$$J_n(-x_p) = -qD_n \frac{dn_p(x)}{dx}\bigg|_{x=-x_p} = q\frac{D_n}{L_n} n_{p0}\left(e^{V/V_T} - 1\right)$$

Total current:

$$I = A(J_p + J_n) = A\left(q\frac{D_p}{L_p}p_{n0} + q\frac{D_n}{L_n}n_{p0}\right)\left(e^{V/V_T} - 1\right)$$

$$= Aqn_i^2\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right)\left(e^{V/V_T} - 1\right)$$

Thus,

$$I = I_s\left(e^{V/V_T} - 1\right)$$

Where,

$$I_s = Aqn_i^2\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right)$$

I-V Curve

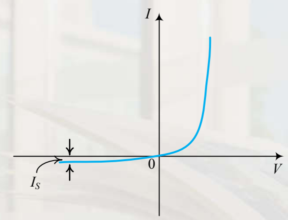

Fig. *pn* junction *I–V* characteristic.

$$I = I_s\left(e^{V/V_T} - 1\right)$$

$$I_s = Aqn_i^2\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right)$$

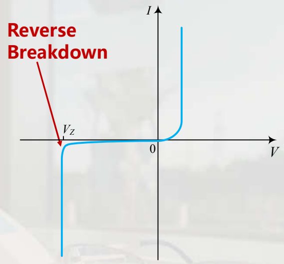

Fig. *I–V* characteristic of the *pn* junction with breakdown region.

Depletion Capacitance (mainly reverse bias)

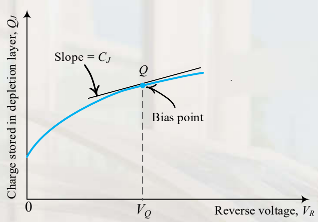

Fig. Charge stored on either side of the depletion layer as a function of V_R.

If $V = -V_R$, junction area A

Total charge in depletion region,

$$Q_j = AqN_Dx_n = AqN_D\frac{N_A}{N_A+N_D}W$$

$$W = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}$$

As **bias voltage** change, **junction charge** change.

This is a **nonlinear** capacitor.

The capacitance is

$$C_j = \frac{dQ_j}{dV_R} = Aq\frac{N_AN_D}{N_A+N_D}\sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}\frac{d}{dV}\sqrt{(V_0+V_R)}$$

$$C_j = A\sqrt{\frac{\varepsilon_s q}{2}\left(\frac{N_AN_D}{N_A+N_D}\right)}\frac{1}{\sqrt{(V_0+V_R)}}$$

Note:

$$C_j = \frac{\varepsilon_s A}{W}$$

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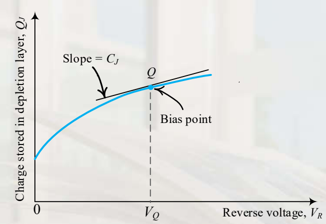


Fig. Charge stored on either side of the depletion layer as a function of V_R.

At zero bias, $V_R = 0$,

$$C_{j0} = A\sqrt{\frac{\varepsilon_s q}{2}\left(\frac{N_AN_D}{N_A+N_D}\right)\frac{1}{\sqrt{V_0}}}$$

Therefore at $V = -V_R$

$$C_j = \frac{C_{j0}}{\sqrt{1 + V_R/V_0}}$$

This is a variable capacitor, controlled by voltage!

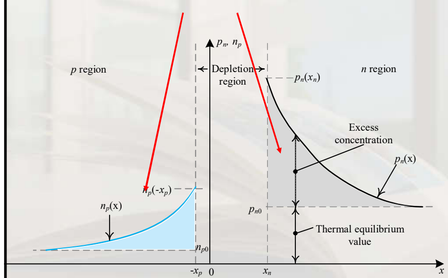
In comparison, for a **linear** capacitor:

$$C = \frac{Q}{V}$$

C is a **constant.**

Diffusion Capacitance (forward bias)

Extra minority carriers stored outside junction under forward bias



$Q_p = Aq \times$ shaded area under $p_n(x)$ curve

Excress concentration $= p_{n0}(e^{V/V_T} - 1) \cdot e^{-\frac{x-x_n}{L_p}}$

$$Q_p = Aq \int_{x_n}^{\infty} p_{n0}\,(e^{V/V_T} - 1) \cdot e^{-\frac{x-x_n}{L_p}}\,dx$$

$$Q_p = AqL_p p_{n0}(e^{V/V_T} - 1) = \frac{L_p^2}{D_p} I_p = \tau_p I_p$$

τ_p has unit of time, its physical meaning is Minority carrier lifetime:

$$\tau_p = \frac{L_p^2}{D_p}$$

Average time it takes for a hole injected into n region to recombine with a majority electron.

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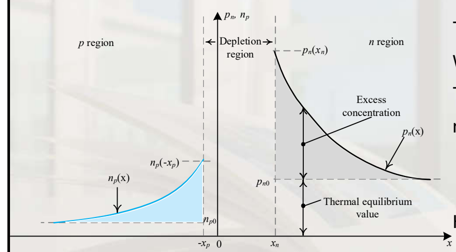


Similarly, $Q_n = \tau_n I_n$

Total charge stored: $Q = \tau_p I_p + \tau_n I_n = \tau_T I$

Where, τ_T is the mean transit time.

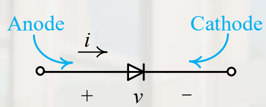
These stored charges correspond to another nonlinear capacitor called **diffusion capacitance**

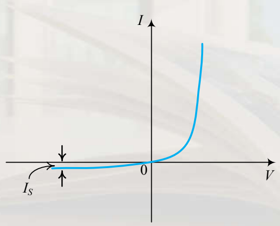
$$C_d = \frac{dQ}{dV} = \frac{d(\tau_T I)}{dV} = \tau_T \frac{dI}{dV}$$

Hence,

$$C_d = \left(\frac{\tau_T}{V_T}\right) I$$

Summary of pn Junction





Built-in potential: $V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$

Depletion Width: $W = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_o - V)}$

<u>Forward bias</u>

I-V curve: $I = I_s(e^{V/V_T} - 1)$

Diffusion capacitance: $C_d = \left(\frac{\tau_T}{V_T}\right) I$

<u>Reverse Bias</u>

Negligible current, $I = -I_s$

Depletion capacitance: $C_j = \frac{C_{j0}}{\sqrt{1+V_R/V_0}}$