EE270: Homework 5

Problem 1

A point-of-load (POL) voltage regulator using a synchronous buck converter is shown in Fig. 1. Losses can be neglected except for the losses due to the inductor resistance R_L and the capacitor equivalent series resistance R_{esr}. The PID compensator is constructed around an op amp. In this problem, you may assume that the op amp has ideal characteristics. The pulse-width modulator has a very large input resistance, so that a voltage injection source $\hat{v}_z$ can be ideally inserted between the compensator and the PWM.

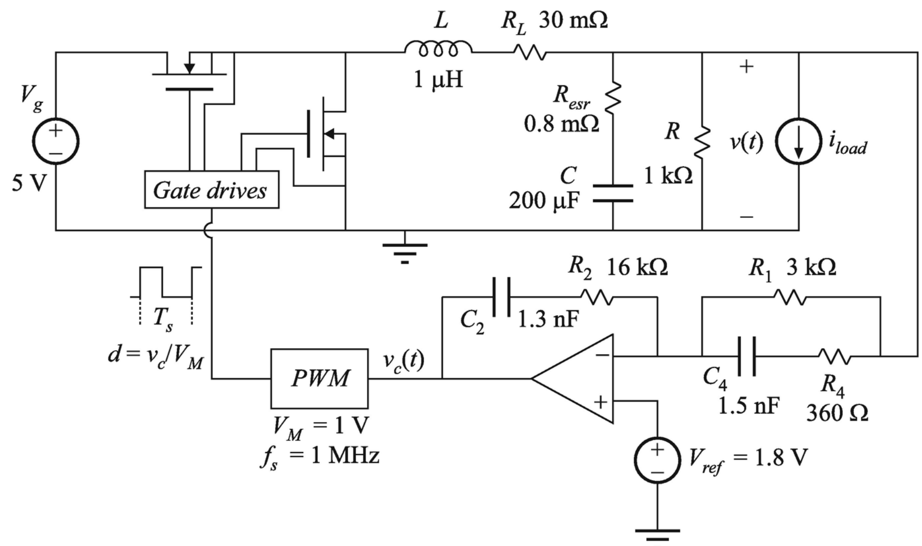

Figure 1: Synchronous buck voltage regulator with a PID compensator.

(a) Derive an expression for the loop gain $T(s)$. The expression should be in standard normalized form. Salient features of $T(s)$ should be expressed in terms of the circuit parameters shown in Fig. 1. Plot the magnitude and phase responses of the loop gain, and determine numerical values for the crossover frequency f_c and the phase margin φ_m.

(b) In this part of the problem, the objective is to determine the closed-loop output impedance $Z_o(s) = -\hat{v}/\hat{i}_{load}$ of the POL voltage regulator using the Feedback Theorem. Derive expressions for $Z_{\infty o}$, Z_{0o}, and the null loop gain T_{nz} in standard normalized forms. Show that the reciprocity relationship holds. Plot the magnitude and phase responses of $Z_o(s)$.

(c) In this part of the problem, the objective is to determine the closed-loop line-to-output transfer function $G_g(s) = \hat{v}/\hat{v}_g$ using the Feedback Theorem. Derive expressions for $G_{\infty g}$, G_{0g}, and the null loop gain T_{ng} in standard normalized forms. Show that the reciprocity relationship holds. Plot the magnitude and phase responses of $G_g(s)$.

(d) In this part of the problem, the objective is to determine the closed-loop reference-to-output response $G_r(s) = \hat{v}/\hat{v}_{ref}$ using the Feedback Theorem. Derive expressions for $G_{\infty r}$, G_{0r}, and the null loop gain T_{nr} in standard normalized forms. Show that the reciprocity relationship holds. Plot the magnitude and phase responses of $G_r(s)$.

Problem 2

Use the circuit averaging method to derive an equivalent circuit that models dc and small-signal ac signals in the Cuk converter. You may assume that the converter operates in the continuous conduction mode, and that all elements are ideal.

(a) Give a time-invariant electrically identical circuit, in which the switching elements are replaced by equivalent voltage and current sources. Define the waveforms of the sources.

(b) Derive a large-signal averaged model for this converter.

(c) Perturb and linearize your circuit model of part (b), to obtain a single equivalent circuit that models dc and small-signal ac signals in the Cuk converter.

Problem 3

In a two-switch PWM converter operating in CCM, the transistor switch absorbs dc power P_{dc} and delivers ac power $P_{ac} = P_{dc}$ to the rest of the circuit. On the other hand, the rectifier switch absorbs P_{ac} from the circuit, and delivers P_{dc}. The converter dc output power Pout can be written in the form

$$P_{out} = P_{direct} + P_{indirect}$$

where $P_{indirect}$ equals the ac power P_{ac} processed by the switches. Reference polarities are selected so that $P_{out} > 0$, $P_{direct} > 0$, $P_{indirect} > 0$. You may assume that losses can be neglected. Derive expressions for the output power P_{out} and for the indirect power $P_{indirect}$ as functions of V_g, I_{load}, and D, and expressions for $P_{indirect}/P_{out}$ and P_{direct}/P_{out} as functions of the dc conversion ratio $M = V/V_g$ for **Buck converter**.

Problem 4

Averaged switch modeling of a flyback converter. The converter of Fig. 2 operates in the discontinuous conduction mode. The two-winding inductor has a $1 : n$ turns ratio and negligible leakage inductance, and can be modeled as an ideal transformer in parallel with primary-side magnetizing inductance L_p.

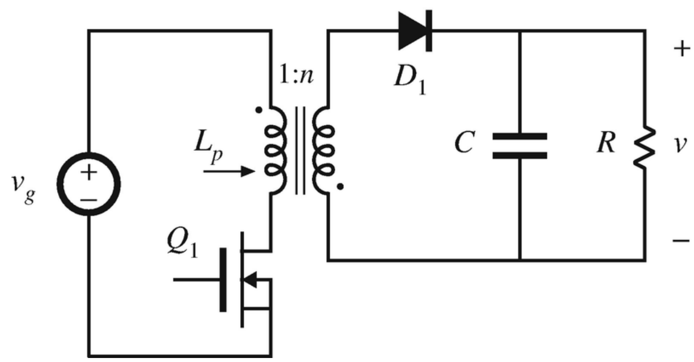

Figure 2: Flyback converter.

(a) Sketch the transistor and diode voltage and current waveforms, and derive expressions for their average values.

(b) Sketch an averaged model for the converter that includes a loss-free resistor network, and give an expression for $R_e(d)$.

(c) Solve your model to determine the voltage ratio V/V_g in the discontinuous conduction mode.

(d) Over what range of load current I is your answer of part (c) valid? Express the DCM boundary in the form $I < I_{cirt}(D, R_e, V_g, n)$.

(e) Derive an expression for the small-signal control-to-output transfer function $G_{vd}(s)$. You may neglect inductor dynamics.